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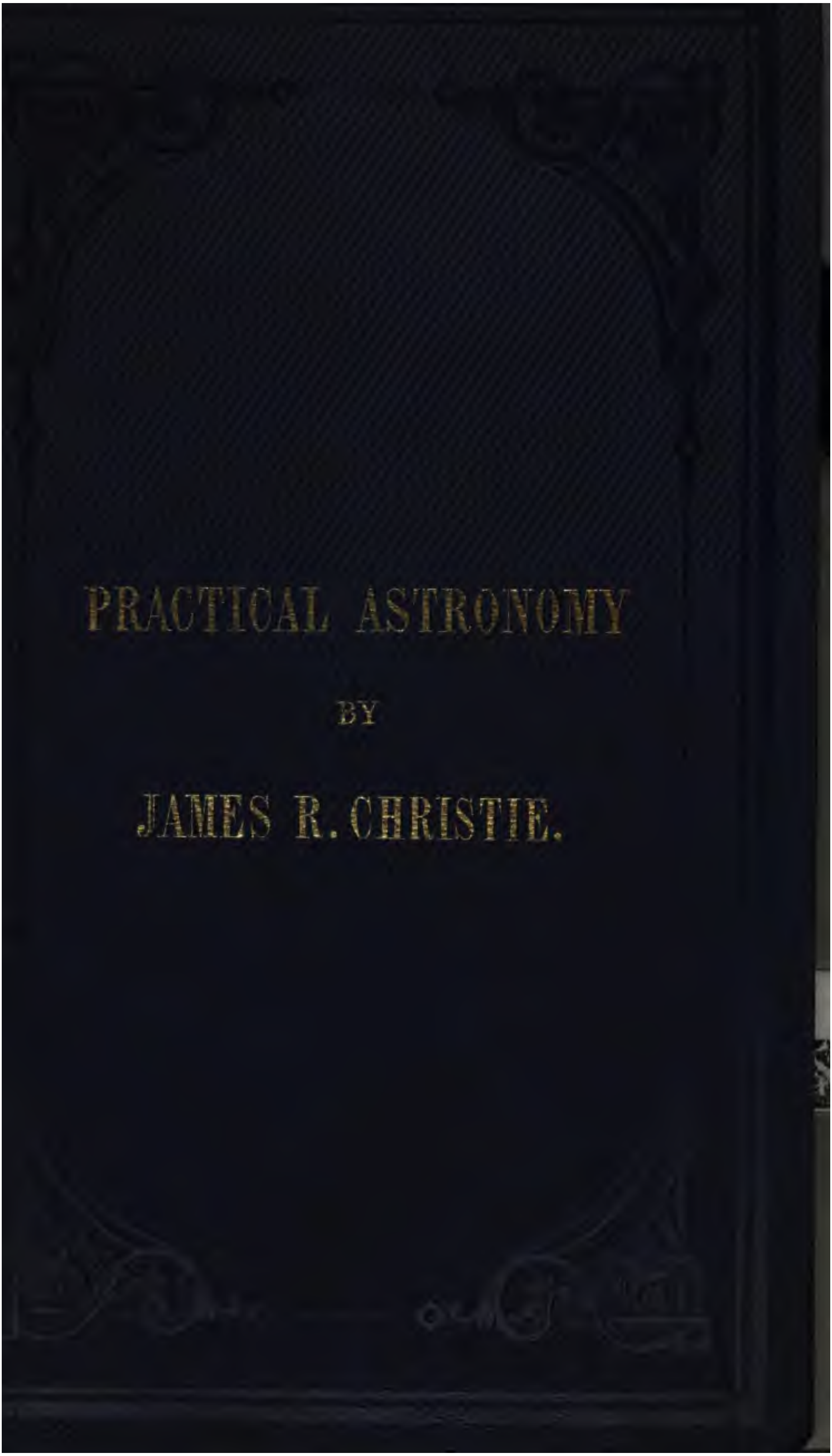

PRACTICAL ASTRONOMY

BY

JAMES R. CHRISTIE.

AN INTRODUCTION

TO

THE ELEMENTS

OF

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PRACTICAL ASTRONOMY.

BY

JAMES R. CHRISTIE, F.R.S.,

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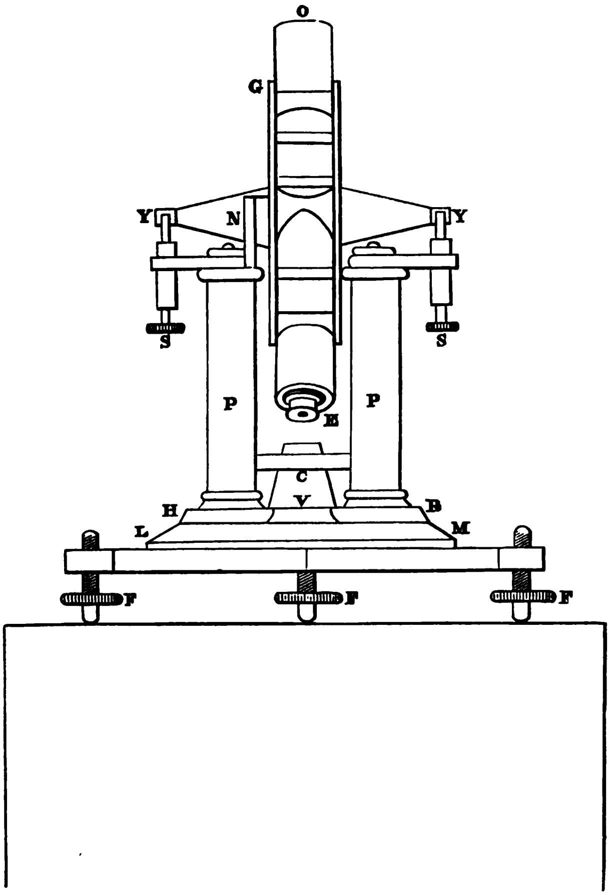
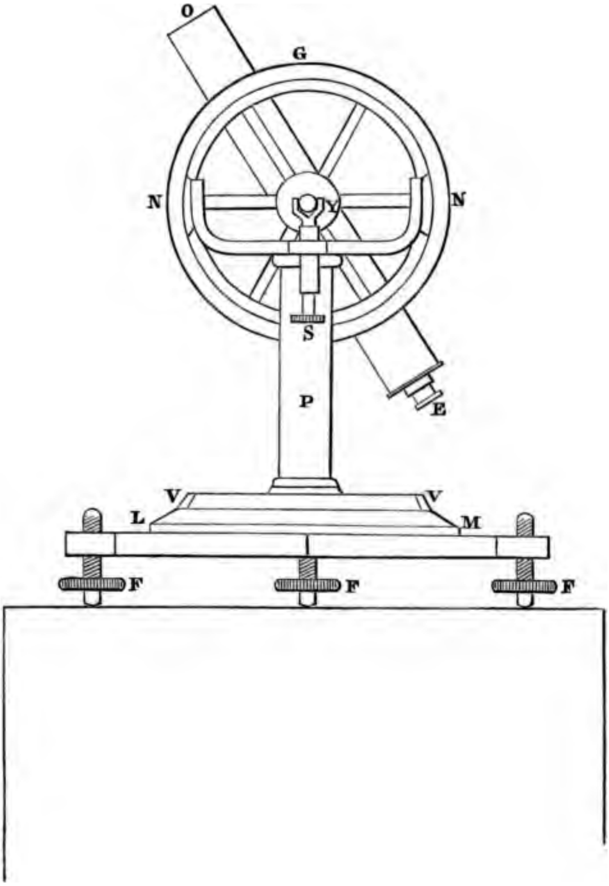


ALTITUDE AND AZIMUTH INSTRUMENT.



ALTITUDE AND AZIMUTH INSTRUMENT.

PREFACE.

THE following pages comprise the substance of a short series of explanatory lectures delivered in the course of tuition to my pupils in the Royal Military Academy. Having myself long felt the want of an elementary work which, without entering far into the details of Physical Astronomy on the one hand, or, on the other, attempting to include within its scope the more delicate questions of instrumental manipulation, would lay a solid theoretical basis for the future progress of the student, and afford at the same time sufficient insight into the fundamental modes of observation to render available the ordinary knowledge of Spherical Trigonometry acquired in our professional schools, I have here attempted to supply the deficiency.

I have, as much as possible, avoided giving specific rules or formulæ for the various forms of problems, preferring to throw the student on his own resources of knowledge of the general principles which apply to each particular case, by rendering it essential for him to refer to the astronomical figure for the elucidation of the means of solution. I believe this to be the best means of eradicating the too common spirit of dependence upon the recollection of formulæ, and of inducing an amount of self-reliance which gives the pupil an important vantage-ground in the prosecution of his further progress.

I have also, on the same principle, preferred, in all cases, to base the operations indicated upon the ordinary applications of trigonometry, rather than make any reference to tables of auxiliary arcs. Even in clearing the lunar distance, the processes are retained in their purely trigonometrical form; and it is a question whether, when a table giving the functions to seconds of arc is at hand, this plan be not, in isolated calculations, preferable to that of employing auxiliary tables, which, besides making the process dependent upon a mere rule, very commonly save the trouble of tabular reference only at the cost of additional risk of error in the application of their arguments.

A set of tabular extracts from the Nautical Almanac is inserted at the end of the volume, in the hope that, besides illustrating the general use of tabulated astronomical data, they will be of service in enabling an instructor to frame questions in the form in which they would present themselves in actual practice.

I have not thought it necessary to make any allusion to corrections for second differences, conceiving it to be better to present the subject, in the first instance, in the simplest possible form, consistent with such accuracy of detail as will prove sufficient to render available observations made with the smaller class of instruments, than to complicate it with corrective formulæ, which, after all, are nearly useless to the young observer.

JAMES R. CHRISTIE.

Woolwich, November 1853.

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INTRODUCTION TO THE ELEMENTS

OF

PRACTICAL ASTRONOMY.

CHAPTER I.

The Solar System.—Form and magnitude of the Earth.—Motion of the Moon. —Satellites of Jupiter.—Eclipses.—Theory of Light.—Reflexion and Refraction.—Motion.—Parallelism of Sidereal Rays.

The study of Practical Astronomy requires a knowledge of the true motions of the heavenly bodies, as distinguished from their apparent motions; though it is from the former satisfactorily accounting for the latter (which can alone be the subject of direct observation) that their claim to truth can be established. The student must therefore, in the outset, consider that the apparently innumerable bodies seen in the sky have, by a series of observations extended over many hundred years, been found to agree in their changes of position and appearance with the assumed fact that certain of them are, in their motions, intimately connected with our earth, while the vast majority have no real motion at all. Generally speaking, those bodies which compose the group or "*system*" of which our earth forms a part are, as we should expect, much more conspicuous than the others; in the little which we need here say respecting them we will consider principally the Sun, the Earth and the Moon, in their relations to that system.

1. The Sun is the great central and most important object in this

group of celestial bodies, which, from their comparative proximity to it and to one another, and from their consequent connexion with it, through the influence of the laws of mutual attraction, is called the SOLAR SYSTEM. The other principal bodies of that system (with certain remarkable exceptions) are called PLANETS, and move round the Sun in paths or "*orbits,*" differing but little in form from circles; these orbits are, in fact, ellipses, whose axes are nearly equal, and which have the sun in one of their foci; but they are subject to a variety of irregularities arising from the mutual attractions of the planets, which are constantly varying with the ceaseless changes in their relative distances from one another, consequent upon their moving round the sun with velocities which, in their turn, are proved to depend upon the relative distances of the bodies from the sun. The names, distances from the sun, &c. of these planets, will be best exhibited in the tabular form:—

Names.	Approximate mean distance from the Sun, in millions of miles.	Periodic time of revolution round the Sun.	Diurnal revolution on axis of rotation.	Inclination of orbit to that of the Earth.
		Days.	h m s	° ′ ″
Mercury	37	87·97	24 5 28	7 0 1
Venus	68	224·70	23 20 54	3 23 32
Earth	95	365·26	24 0 0	0 0 0
Mars	144	686·98	24 39 22	1 51 4
Flora	209	1193·30	....	5 53 2
Victoria	221	1303·20	....	....
Vesta	225	1325·21	....	7 8 26
Iris	226	1341·60	....	5 28 16
Metis	227	1345·90	....	5 35 36
Hebe	230	1379·80	....	14 46 42
Parthenope...	230	1379·80	....	....
Psyche	Discovered 1852; belonging to the same group as those between Mars and Jupiter, which are, like them, very considerably smaller than the other planets.			
Thetis				
Melpomene...				
Fortuna				
Massilia				
Lutetia				
Calliope				
Thalia				
Astræa	245	1511·00	....	5 19 25
Egeria.........	245	1513·00	....	....
Irene	247	1517·40	....	....
Eunomia......	251	1573·60	....	....
Juno	254	1591·00	....	13 3 26
Ceres	263	1681·54	....	10 37 12
Pallas	265	1681·71	....	34 37 16
Hygeia	297	2024·10	....	3 47 5
Jupiter	490	4332·60	9 55 37	1 18 51
Saturn.........	900	10758·97	10 16 2	2 29 35
Uranus	1800	30688·71	....	0 46 26
Neptune	2820	60129 00	....	1 47 1

2. In this system, the Earth, being that body which we inhabit, is the one whose form and constitution is of most importance to us, but for our present purpose it will suffice to confine our attention to its form. It is found by observation to be universally true of all the other planets, that they approximate in shape very closely to spheres, and we might, upon such ground alone, consider it as next to certain that the Earth also is of that form; but when we moreover find that such an hypothesis is supported by the observable convexity of the surface in all directions, at any point where our horizon boundary is sufficiently distant and uninterrupted by local inequalities to afford the means of observation, as in extensive plains and, more decidedly, on the surface of the sea, we can have no doubt left that our planet differs but little from the spherical form. By careful and elaborate modes of measurement, this form is found to approximate most nearly to the solid which would be generated by the revolution of an ellipse about its minor axis; the ratio of the two axes being very nearly 298 to 299.

3. As the Moon, next to the Sun, at once attracts our notice both on account of its apparent magnitude and the remarkable changes which its shining surface undergoes from day to day; so, from its proved connexion with the Earth as the centre round which it constantly revolves, attending, as it were, the latter in her revolutions round the Sun, and from its importance in some of the practical applications to which our knowledge of its motions will hereafter be applied, it is, next to the Earth and the Sun, the body of most consequence to us to study. The Moon, then, revolves round the Earth in a period of 27 *days* 7 *hours* 43 *minutes* and 11·5 *seconds*, her orbit being, as with the primary planets, slightly elliptical, and the direction of her motion being from west to east. Ordinary observation teaches us that she at all times presents the same portion of her surface towards the Earth, and it would follow as a necessary consequence, that she must make one revolution upon an axis at right angles to the plane of her orbit in the same interval as that in which she makes one revolution round the Earth; because, had she no rotatory motion on an axis, she would, as she revolved round the Earth, bring every point of her surface successively into our view, as she also must, in the course of time, did her periodic time, in the smallest degree, constantly exceed or constantly fall short of the time of her making one revolution on her axis.

4. This effect is, in fact, produced, in a modified form, as a result of the unequal angular motion of this body round the earth, owing to the form of her orbit, which by the laws of elliptic motion causes her in equal intervals of time to pass over unequal portions of her course; and it is found that when her motion is slowest, a small portion of her eastern border comes into view, and a corresponding portion of her western border disappears; while, at the part of her course where her motion is quickest, the contrary effect takes place. It is found also that when the upper and lower borders of the moon's visible surface are carefully observed, these also undergo a similar change, which is attributable to her axis of rotation not being exactly perpendicular to the plane of her orbit. These effects are called the "*Librations*" of the moon; they are alluded to here simply as illustrative of the fact of the moon's rotation upon her axis, which, when her orbital motion round the earth is admitted, must be plain to the most ordinary observer of the distinctive marks upon her surface.

5. The rotation of the moon upon her axis being thus clearly established from the single datum of her revolution round the earth, it is only natural to conjecture that other, if not all the bodies connected with the Solar System have a similar motion upon axes which are, severally, more or less inclined to the planes of their respective orbits. In some, this motion is made visible to us by the aid of powerful instruments, and we may very safely infer from these instances, that all are subject to rotatory motion; but as in no other instance besides that of the moon is such rotation found to coincide in interval with the periodic time of orbital revolution, we must consider that her case is peculiar, and that the interval of rotation has probably no connection with the periodic time.

6. There appears then every probability that, among the rest, our earth, besides her annual motion in her orbit, rotates upon an axis; if, therefore, we can show that certain appearances in the heavens may be explained upon two distinct hypotheses, one of which is in itself highly improbable, and the other is simply that of the earth's rotation upon an axis inclined at a certain angle to the plane of her orbit, there can be no uncertainty as to which we shall choose to give our assent to. We shall in the next chapter enter more in detail into this point; at present we will content ourselves with

establishing the probability of that motion of the moon round the earth which we have assumed as the basis of our reasoning, by reference to the planet Jupiter, round which a similar arrangement, though on a much more extended scale, is going on before our eyes.

7. This planet is, by the aid of a telescope, seen to be attended by no less than *four "satellites"* or moons, which revolve around it, according to the same law as that already stated for the planets around the sun. A close observation of these satellites brings under our notice a point of resemblance to our moon, which, while it excites our admiration, is also of importance to us to remark upon here, as we shall have to refer to it in our future practical processes. It is observed that they from time to time suddenly disappear, and, after a short interval, reappear as suddenly; and it is found that this effect takes place when the particular satellite to which it occurs is on that side of Jupiter which is opposite to the sun, and it is thence inferred that the satellite, shining only by light received from the sun, becomes darkened, and therefore invisible when that light is intercepted from it by the body of the planet; the satellite, in fact, passes into the *shadow* of Jupiter, and remains obscured until, by a continuance of its motion, it again emerges into the sun-light beyond the shadow. As the motion continues, the satellite gradually passes round to that side of the planet which is presented towards the sun, and we then observe the converse phenomenon of the planet eclipsed by the satellite; but, inasmuch as the diameter of the largest of the satellites is less than one twentieth of that of Jupiter, the entire shadow of the satellite is seen as a dark spot upon the planet. It is clear then, that, if we could transport ourselves to the surface of the planet and place ourselves in that dark spot, no ray of sun-light could reach us, until, by the combined effect of the motion of the satellite and of the rotation of the planet, we should find ourselves beyond the boundaries of the shadow, in the act of passing out of which we should, were we to direct our attention towards the sun, see it gradually emerge from behind the satellite.

8. This effect is from time to time witnessed by us in our own lunar system, when the moon, being at or near her closest proximity to the earth, has an angular diameter greater than that of the sun, and the centres of the three bodies are exactly, or very nearly, in

the same straight line; but, when that relation of the angular diameters is reversed, which happens when the moon is at or near her greatest distance from the earth, the body of the moon not being of sufficient apparent magnitude to exclude the whole of the sun's light, leaves a ring of brilliant light round the edge of the sun, and produces what is called an "*Annular Eclipse,*" in contradistinction to the "*Total Eclipse,*" which takes place under the former circumstances.

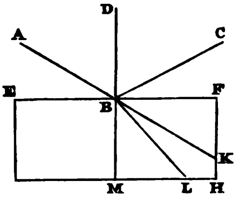
9. The eclipses of Jupiter's satellites are, like all the planetary motions, made the subject of very exact calculation, and can be predicted with a great degree of accuracy. When, however, this calculation was first attempted, there appeared some very remarkable discrepancies, which seemed to depend upon the relative positions of Jupiter and the Earth; since, when these planets were at those points of their respective orbits most distant from one another, the eclipses were observed to take place *after* the calculated time; while, when they were at their greatest proximity, they took place *before*. These results were clearly accounted for, on the supposition that the light of the satellite, though actually extinguished, as it were, by its immersion in the shadow of Jupiter, still continued to be transmitted to the earth; or, which amounts to the same thing, that the last shining effect of the satellite, instead of being instantaneously transmitted to the earth, took a certain interval of time to pass through the distance, between the two planets. This interval was found to be 16 *minutes* 26·6 *seconds* for a distance equal to one diameter of the earth's orbit. This hypothesis of the effect of light requiring a definite interval of time for its propagation from point to point, small though that interval be, considering the enormous distances with which we have to deal in our astronomical inquiries, is further corroborated by an observed change in the apparent positions of all the heavenly bodies, which is also in strict accordance with what would result from the truth of the principle. This last result will be again referred to and illustrated; but, for the present, we will turn our attention to some other of the properties of light.

10. The generally received theory of the propagation of light is, that it takes place by a series of oscillations to and fro, of the particles of the medium or substance through which the light passes; these oscillations being transmitted from particle to particle, and

continuing as long as the exciting cause continues to operate; so that there is no transmission of any material particle from the point of origin of the light to the point where its effect is observed, but only a transmission of this oscillatory motion, which is propagated in all directions from the point of its origin; just as, when a stone is dropped into a pond of still water, the ripple produced is propagated in a series of concentric circles, more and more distant from the source of disturbance, but the particles of the water are so far unaffected, that they occupy, when the effect has ceased, precisely the same relative positions as they did before it began; though the small waves which form the ripple may be traced some hundreds of feet distant from their origin. It is this close analogy to the mode of propagation of wave-motion in water, which has given to this hypothesis the title of "*The Undulatory Theory of Light.*"

11. In accordance with this theory, the term "*ray of light*" must be understood to have the same meaning as regards light, that the term *wave* has with reference to the surf which beats upon the sea-shore; and as, in the latter case, a wave, when once generated, will proceed in the right line of its original motion until interrupted by some change of circumstances at the point it has reached, so, the "*ray*" will maintain its direction as long as the arrangement of particles, by means of which it is transmitted, remains unaltered: in fact, whatever change is known to take place with respect to the *wave* under certain circumstances, the same, or at least an analogous change, may be safely inferred to result from a corresponding arrangement with regard to the *ray*. Thus, when a ray meets in its course a material plane surface, brightly polished, it is (just as the wave would be) turned aside or "*reflected*" from its original course, at a greater or lesser angle, according to the angle at which the surface is inclined to that original course; and the law in accordance with which this "*reflexion*" takes place is found to be the same in both, viz. that the angle which the original course makes with the perpendicular to the plane at the point of meeting, is the same as that which the reflected course makes with that perpendicular: or, calling the first of these angles the "*angle of incidence,*" and the latter the "*angle of reflexion,*" the law is simply stated thus:—"*the angle of incidence is equal to the angle of reflexion.*"

Fig. 1.



12. Thus if AB were the course, in air, of a ray of light, which impinged at B upon the plane surface EF of a material body; then if BD be perpendicular to EF, and we make the angle DBC equal to the angle ABD, this latter is the "*angle of incidence*"; DBC, its equal, is the "*angle of reflexion*"; and BC is the course which the ray would pursue after impinging at B.

13. In the above illustration, we have merely considered the course which the ray would pursue after impinging at B as if the whole effect were transmitted unimpaired to C; but ordinary experience tells us that if the substance EH be what we denominate "*transparent,*" a part, at least, of the light will be propagated *through* the substance, and the question therefore remains, what course does this remaining portion of the ray pursue?

14. As we may consider that the particles of the substance EH are not so readily set in that kind of motion which results in light as those of the air from A to B, and as, in our figure, the lower part of the last particle of air which is set in motion is in contact with the surface EF, it would seem probable that the last particle of air would have a tendency to turn upon its point of contact with the surface, and thus, instead of transmitting the wave-motion in the original direction ABK, cause it to take a new direction BL, *nearer,* that is, to the perpendicular BM, than it would if the body EH had not been interposed. This is, in fact, the result which does take place, and the effect is known by the term "*refraction.*" The angle MBL is the "*angle of refraction,*" and the ratio of the sine of the angle of incidence to the sine of the angle of refraction is found to be constantly the same for the same substance, when the medium through which the ray originally moves is the same.

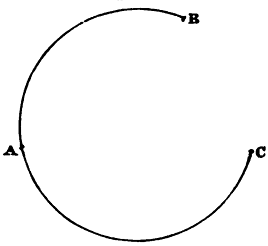
15. If the ray were proceeding in the contrary direction, that is, from L towards B, the contrary effect would take place, viz. the ray would be refracted in the direction BA, further from the perpendicular BD than the direction of its original course; so that in proceeding from any medium into another which is more *dense,* the ray is refracted *towards* the perpendicular; while in proceeding from any medium into another which is more *rare,* the ray is refracted *away from* the perpendicular. Supposing that the medium

through which the ray passes be more and more dense in the direction of the perpendicular BM, then the ray must necessarily be more and more deflected from its original course, and it will consequently describe a line which becomes more and more *curved*; the concavity being turned towards the perpendicular BM.

Having now stated a few fundamental facts, and established certain principles which will be of use to us hereafter, we will, in the next chapter, resume the consideration of the apparent and real motions of the heavenly bodies. But before proceeding with this part of our subject, it will be well to point out the nature of such an inquiry, and to illustrate the kind of ambiguity which always more or less besets it.

16. A little consideration will show that, in applying the term "*motion*," we necessarily assume the existence of at least three material points in space, any one of which may be taken as the subject of motion; let us for instance assume that the three material points A, B, C are the only ones existing in space, and that it is a question whether the point A "*moves*"; then, as regards any observation made from that point, it is plain that, supposing B and C to remain stationary, the only change which can be observed at A (assuming that we have no means of measuring the distances AB, AC) is that of the angle BAC: but this angle will remain unaltered as long as A moves in the circumference of the circle described through the three points, because it will then always be in the same segment of that circle; consequently, whether A move in such circular course, or remain stationary, the effect, *at A*, is precisely the same, unless that point come, as it were, into actual contact with B or C. If now we discard C, we at once perceive that A may move in any manner as regards the remaining point B, without affording, *at A*, any indication of the fact, until, as before, it come into actual contact; and, if the point B be also annihilated, we see that the detection of any motion in the sole existing particle A is impossible, so that, in such a case, the question of *motion* or *rest* has no meaning.

Fig. 2.



17. The same result occurs if we assume that the triangle ABC,

constituted by joining the three points, have a motion through space; for the positions of A, B, and C (with respect to one another) remaining unaltered, the motion or rest of the system of bodies A, B, and C is quite indeterminate.

18. Now let us suppose that the angle BAC, *as observed at A,* has changed. This change may be the result of three distinct forms of motion, or of all combined. *First,* A may have moved, so as no longer to be in the circumference of the circle described through the three points. *Secondly,* B may have moved to some position not in the direction of the line AB. *Thirdly,* C may have moved to some position not in the direction of the line AC. In any case A is said to have moved "*relatively*" to B and C, or B and C are said to have moved "*relatively*" to one another; it being understood that A is in either case considered as the point at which the change of angle is observed.

19. If, then, we know, from considerations unconnected with B and C, that the point A is constantly altering its position, and that it is in no respect constrained to move in the circle described through the three points, and we observe that the angle BAC undergoes no sensible change, we are compelled to consider either that the system A, B and C (that is the triangle ABC) has the same motion as A (which is what we mean, when we say that A, B and C are "*relatively at rest*"), or that the distances of B and C from A are so great, that the change of place of A, though we know it to be considerable, has, when compared to these distances, a magnitude which is insufficient to produce any change in the angle BAC, capable of measurement by the most delicate instruments we can construct.

20. It is upon evidence such as this that the immensity of the distance of much the largest portion of the heavenly bodies rests. Our earth is known to move over an orbit, the length of whose major axis is more than 190 *millions of miles,* and yet, in but two or three cases, is any change due to the mere alteration of the earth's place in her orbit, in the angular distance between the stars (which we, for this reason, call "*fixed*"), even suspected.

21. Since, then, the whole orbit of the earth is too small to produce any positive change in the angular distances of the fixed stars from one another, it must evidently be treated as a mere point when viewed from them; and hence we may consider that all rays of

light coming to us from any one of them, trace (as far as our senses are concerned) the same course, whatever position we occupy; or, which amounts to the same thing, that all rays coming from any one of them to us in any part of the earth, and in any part of our orbit, take directions which may be treated as parallel to one another.

We shall proceed in the following chapter to apply these various principles to the illustration of those details which are most intimately connected with the study of practical astronomy.

CHAPTER II.

Appearance of the Heavens.—Measurement of altitude and of azimuth.—Apparent diurnal motion of the Stars.—Diurnal rotation of the Earth.—Relative motion of the Earth and the Sun.—Parallax.—Aberration.—Refraction.

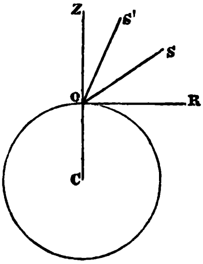
22. If, on a bright night, a spectator place himself in the open air, in a position which affords an unobstructed view of the whole sky, he cannot fail to remark that the brilliant specks of light with which it is studded appear to be either placed upon or enclosed within the comparatively dark hemispherical background which we call sky, and, if he be aware of the spherical form of the earth, he will be too apt to consider that there is some necessary connexion between it and the hemispherical vault which he finds spread above and around him. But it is not so; this remarkable appearance of form in the sky arises solely from the uniformity of its colour and the great distance to which we necessarily refer every point in it, since it appears to enclose every body evident to our sight: these two facts have the effect of preventing any one point in the sky appearing nearer than another, and consequently producing the hemispherical appearance noticed.

23. If the spectator now turn his attention to the stars with which this vast vault is studded, he will find that they are in constant motion; some continually increasing in apparent height above the terrestrial objects around him; others decreasing; the former, if watched for a sufficient length of time, will be found, after attaining a certain elevation, also to commence a similar apparent descent, until they, as well as those others, either are hidden by the visible horizon, or again commence rising. In the course of these changes he will notice that, whatever star he may select for more particular observation, it will, as time passes, appear to describe a curved path, which, as far as he can judge, is a circle; and still closer attention will convince him that, each star having its individual path so described, the circles become less and less as the stars selected for notice appear to be near a certain point in the sky, until, at length,

he observes one star in particular which describes a circle so small, that, to his unassisted vision, it appears quite stationary. Now assuming that these stars are infinitely distant, let us see whether we cannot trace a connexion between the above observed changes of their position and the fact of the earth revolving upon an axis whose position we may assume for the present to be fixed.

24. But before we do this, it will be desirable to acquire clear ideas concerning the true meaning of the terms which we employ to indicate change of position as regards objects external to the earth. When we speak of one star being more elevated than another, we mean that the ray of light emanating from that star and impinging upon the point of observation, makes a greater angle with the plane which is tangential to the surface of the earth at the point whereon we stand, than the ray of light emanating from the other star makes with the same plane. These angles are necessarily complementary to those which the same rays make with a line drawn through the point of observation perpendicular to the tangent plane, and which being produced "*downwards*" passes through the centre of the earth. Thus if O be a point on the surface of the earth and OR a tangent to the surface, supposed to be drawn successively in the vertical planes* which contain the rays S′O, SO respectively, then SOR, S′OR are the angles of elevation, or, as we shall in future call them, the "*altitudes*" of the stars S and S′; and these angles are the complements of the angles SOZ, S′OZ made by the rays SO, SO′ with the line COZ drawn through the centre of the earth and the point O.

Fig. 3.



25. Again, a star, besides being more elevated than another, may have a position either on one side or the other of it, and our ideas on this subject will become much clearer if some definite point in the horizon be taken as a general point of reference as regards lateral position. If we take the point of the horizon which is in the same vertical plane at the fixed point round which, as we have seen, all the stars appear to revolve, as our point of reference, and measure the angular distance on the horizon between this point and the vertical plane passing through the star, we have at once a

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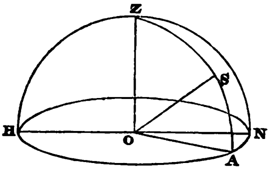
* A vertical plane is any plane passing through the straight line which joins the place of the observer and the centre of the earth.

Fig. 4.

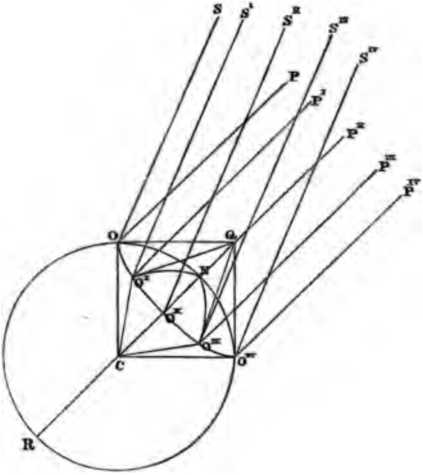


means of determining the horizontal position of that star and of comparing such position with that of any other body: this angular horizontal distance is called the "*Azimuth*" of the star. Thus if NAH represent the visible horizon of the spectator at O; OZ the vertical line perpendicular to the plane NAH; SO the direction of the ray of light from the star; the plane ZOAS will be perpendicular to NAH; and N being the point of the horizon mentioned above, NOA, its equal NZA, or the arc NA, will be the azimuth of star. The point Z in which the radius CO produced may be supposed to meet the hemisphere over head is called the "*Zenith*" of the place of observation, and is employed to mark the common origin of the angular distances ZOS, ZOS', &c. in fig. 3, which are called the "*Zenith distances*" of the stars S, S', &c., and are the complements of the "*altitudes*" SOR, $S'OR$, &c.

26. Having premised thus much, let NOR (fig. 5) represent the earth, of which C is the centre; NCR the axis of rotation assumed to be in the direction of the line which joins the centre of the earth and that point in the heavens round which the fixed stars appear to revolve; O, O', O'', O''', O^{IV} the successive positions of a point O during one half-revolution of the earth upon her axis; PO, $P'O'$, $P''O''$, $P'''O'''$, $P^{IV}O^{IV}$ the rays of light parallel to the axis of the earth, coming from a star which we may assume to be in the line of prolongation of the earth's axis (art. 21.); SO, $S'O'$, $S''O''$, $S'''O'''$, $S^{IV}O^{IV}$ rays of light coming from some star in a different direction from the former. Then N being one of the assumed "*poles*" of the earth, that is one of the extremities of the earth's axis of rotation, the lines ON, $O'N$, $O''N$, &c. will indicate the constant direction of a distant object in the visible horizon of the observer, in the same vertical plane as the imaginary star P; and the planes NOC, $NO'C$, &c. will be the different positions which the vertical plane passing through O and P successively takes as the earth revolves. Let the tangents OQ, $O'Q'$, $O''Q''$, &c. be drawn to meet the axis produced in Q; then since the angles OQC, $O'QC$, $O''QC$, $O'''QC$, $O^{IV}QC$ are evidently all equal, and PO, PO', PO''', PO^{IV} are all parallel to QCR, the angles POQ, $PO'Q$, $PO''Q$, $PO'''Q$, $PO^{IV}Q$ are all equal.

27. Again, since QC, OC and QO are all in one plane, and QC and PO are parallel, it follows that PO is in the vertical plane QOC; and the same at the points O′, O″, &c. Now the angles POQ, &c. are the angles of elevation or "*altitudes*" of an imaginary star, the rays from which are PO, P′O′, &c.; it follows therefore from the first part of the demonstration that the elevation of such star would always be the same, and from the latter that it would always appear in the same lateral direction, or have the same "*azimuth*," which will, in fact, be 0°, 0′, 0″ in all positions of the earth.

Fig. 5.



28. Let us now turn our attention to the rays SO, S′O′, &c., and we shall find that inasmuch as these rays, and also the rays PO, P′O′, &c. are parallel, the planes POS, P′O′S′, &c. are parallel, and therefore make an angle with the vertical planes COQ, CO′Q, &c., which is continually changing, since these last are not parallel; but the lines PO, P′O′, &c. have been proved to have always the same position as regards the lines OQ, O′Q, &c.; it follows then that the direction in which the star S is seen is continually altering; moreover the stars S and P are always at the same angular distance from one another (art. 20.); consequently, the star S must appear to describe a circle about the imaginary point from which we have assumed the rays PO, P′O′, &c. to emanate, and which, being the

point where the line joining the poles of the earth cut the celestial hemisphere is called *"the pole of the heavens."*

29. If we assume that at O the lines SO, PO, QR are all in the same plane, it is manifest that they cannot remain so, since the plane passing through OP and QR is continually altering its position, while that through PO and SO remains parallel to itself; but, when the point O has arrived at O^{IV}, these lines are again all in one plane, the only change which has taken place being that S^{IV} is now seen between P^{IV} and the horizon, whereas at O, P was seen between S and the horizon, the angles POS being in both cases the same. S must therefore have apparently described a semicircle about P as a centre, while the point O has been revolving from O to O^{IV}; and it is clear that, the same process continuing in the same direction, S will apparently describe the remaining semicircle while the point of observation is describing the remaining portion of its course (on the further side of the sphere) necessary to bring it round to O.

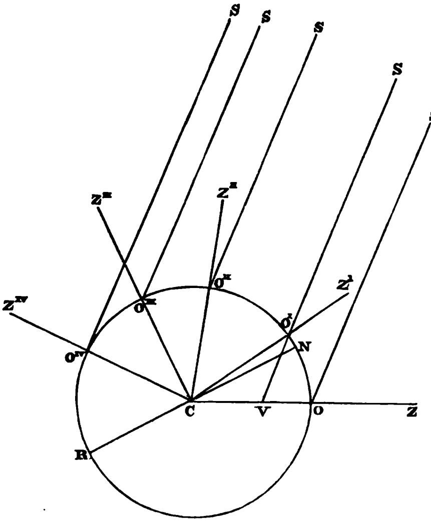
30. It appears then that every fixed star not situated exactly in the prolongation of the earth's axis of rotation, must, on the hypothesis of that rotation, apparently perform a circular course around that point, once in the interval of time which elapses during one complete revolution of the earth on her axis, which is found to agree with the results of observation; and as we have already shown the great probability of the earth's having, like other planets, a rotatory motion of this kind, we may now consider that the existence of such a motion is satisfactorily proved.

31. It is evident that the various positions of any, the same, star will accurately indicate on its particular circle the amount of change which has taken place in the position of an observer on the surface of the earth, which change may, in fact, be due either to her motion on her axis, or combined with this, to an actual change in the position of the observer by his own locomotion in a small circle whose plane is at right angles to the axis of the earth.

32. Let us now see what will be the effect of supposing a change of position in the place of the spectator, in a direction at right angles to that which it undergoes through the rotation of the earth.

Let O, O', O'', O''', O^{IV} be different places on the same great circle whose plane passes through the axis of rotation NR, and let SO, SO', SO'', &c. be the parallel rays of light emanating from a star S situated in the plane of the circle NOR; and let OZ, $O'Z'$, $O''Z''$, &c. be the directions of the zeniths of the several places; then, we see, that the "zenith distance" ZOS of the star S is different at every successive station; and that between O' and O'' the measurement of the zenith distance changes its direction as regards the position of the zenith.

Fig. 6.



33. Not only, however, is the change in these zenith distances distinctly indicative of a change in the position of the observer on the circle NOR, but the amount of that change is readily derivable in the simplest manner from those distances; for if we produce the line SO' to meet CO in V, it is at once seen that the angle OCO', which measures that amount of change, is equal to the difference between SVO and $VO'C$, that is, between the zenith distances SOZ and $SO'Z'$.

34. We have now, therefore, established the very important fact, that the appearances in the heavens, which we may be enabled to observe with a greater or less degree of accuracy, are so intimately connected with the changes of position which we may undergo, either voluntarily, in travelling, or involuntarily by the continual rotation of the earth, that these latter may very well be considered

to be capable of accurate determination as soon as we are in possession of the means of assigning precise values to the former: to show how these values are practically determined, and then to illustrate more in detail than we have done the theory of their application to the determination of the amount of change in the geographical position of the observer in any particular case, is the principal object which we have in view.

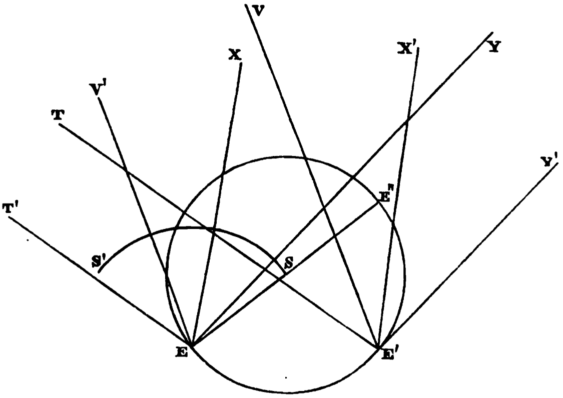
ON PARALLAX.

35. Having shown that what is at any time happening, as regards change of position on the earth, is faithfully chronicled for our perusal in the sky, it will tend greatly to simplicity of conception, if we, in future, treat the record as if it were the event itself; that is, if we consider the apparent motion of the stars as real, and look upon the earth as at perfect rest. From what we have said, there can be no doubt that any results at which we may arrive on the one hypothesis must necessarily be borne out by the other; the real fact is, that the rotation of the earth upon her axis is so far incapable of strict proof, that, as we have before hinted, its truth may be said to rest mainly upon the absurdity involved in the contrary hypothesis of supposing the whole of the stars (which are known to be immensely distant) to revolve round a body utterly insignificant in size compared to them, and which can itself be shown to have a motion in space round a central body, the sun, which is probably one of their own number.

36. It will greatly facilitate our future progress if we adopt the same artificial conception of the changes which arise in the visible heavens from the proper motion of the earth in her orbit round the sun, and view these changes as really what they seem to our senses to be, viz. a variety of motions of the sun, moon and planets among the other stars, which, from their permanence of relative position, we call "*fixed.*" To show that in this case the appearances are identical on the two hypotheses of the earth moving round the sun, and of the sun moving round the earth, let S represent the position of the sun and E, E′ two positions of the earth in her nearly circular orbit; and let TE, VE, WE, XE, YE be the directions of certain fixed stars with which the position of the sun is to be compared: T′E′, V′E′, W′E′, X′E′, Y′E′ the directions (the same in fact as before) in which those stars are seen when the earth is

at E′; then it may be observed, that, the earth being at E, the sun is seen nearly in the direction of the star Y; but, when the earth has arrived at E′, the sun will appear nearly in the direction

Fig. 7.



E′T′, that is, nearly in a line with the star T; and it follows that in the intermediate positions between E and E′ the sun must have appeared successively in the directions of the stars X, W, V, T, and must have performed an apparent course SS′, which precisely corresponds to the actual course of the earth from E to E′, since the distance between the two bodies E and S is entirely unaffected by the apparent change in the direction of the sun. When the earth arrives at the opposite point of its orbit E″, it is evident that the sun, which at E appeared in the direction EE″, will, at E″, appear in the directly contrary position E″E, and will have therefore appeared to make a semi-revolution about E.

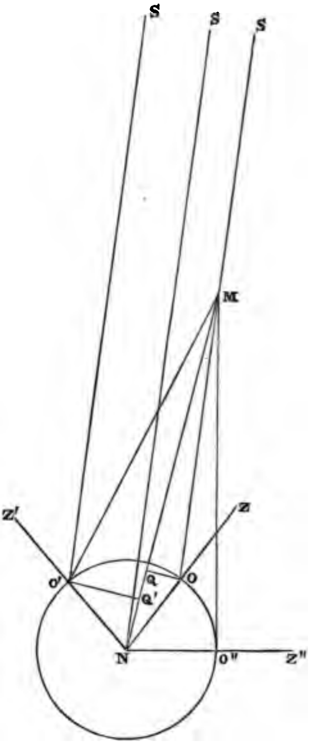
37. It seems then that it must be of the utmost importance to study the effect which the movements of the earth have upon the apparent positions of the other bodies, which will form the subjects of our observation; for the preceding illustration would indicate that it is the proximity of the sun to our planet which causes the reciprocal connexion of the changes which we noted, which indeed would have been the same had there been no connexion between the sun and earth; had these bodies in fact been viewed as existing, one or both in a state of motion, in a certain proximity to one another, compared to the infinite distance of the fixed stars.

38. We may fairly anticipate then that the diurnal motion of the

earth upon her axis will not have precisely the same effect upon the apparent position of the sun, moon and planets which it has upon the fixed stars; that is, added to the apparent motions of those nearer bodies, which they have in common with the more distant, there will be an apparent displacement of the former among the latter, due to their greater proximity.

39. In order to show that this is really the case, let N represent one of the poles of the earth; O and O′ two positions of the same place as the earth revolves; OZ, O′Z′ the directions of the zeniths of those places; SO, SO′ the parallel rays of light coming from a fixed star in the same plane ZO O′Z′; and M the position of some body near the earth, say the moon, also in the same plane. Then at O, the zenith distance of the star, as well as of the moon, is ZOS; and, at O′, the angle Z′O′S measures the zenith distance of S, while Z′O′M measures that of M; so that, quite independently of any change in the actual position of the moon, its apparent place among the stars has altered, owing to the place of the observer having been transferred from O to O′ by the rotation of the earth.

Fig. 8.



40. This change of place among the fixed stars is measured by the angle SO′M, which is the difference of the zenith distances of the star and the moon, or by its equal O′MO, which is the angle subtended at the moon by the arc OO′ described by the place of observation. Now this arc OO′, which is the sole cause of the incongruity we have noticed, can only be got rid of by assuming the observations to have been made at N, the centre of the earth, where its length becomes nothing; and we see, that, in this position, whether the zenith of the supposed observer be in the direction NZ or NZ′, the angular distance of the moon from the

star is the constant angle SNM, though the zenith distances of the star in the two positions are ZNS, and Z′NS the same as ZOS and Z′O′S, which they were before. Now the angle SNM, which is the angular difference between the position of the body at M when seen from O, and that which it has when seen from N, is called the *"Parallax"* of that body, and, being equal to the angle NMO, it may be defined as *"the angle subtended at the body by that radius of the earth which passes through the place of observation."* Its value must evidently be subtracted from the zenith distance as observed at O, viz. ZOM, in order to obtain its zenith distance as seen at the same instant from N, viz. ZNM.

41. It is a matter of importance to determine the greatest value which this *"Parallax"* can have as regards any particular body, and by considering the triangle NMO, we see that the radius NO will subtend the greatest angle at M when NO″ is perpendicular to O″M: the angle NMO″, then, measures what is called the *"horizontal parallax"* of M, because the body M has a zenith distance of 90°, and is therefore in the horizon of the observer. In all other positions the amount of parallax is smaller and smaller as the zenith distance of the body diminishes, until, at last, when it becomes 0, that is, when the body is in the zenith of the observer, it is seen in the direction of the radius produced, and the angle subtended by that radius becomes 0 likewise.

42. The amount of parallax of a body is evidently connected with its distance from the earth; and if we refer to the last figure, we shall see that NO″ = NM sin NMO″, or, if we call the horizontal parallax H, the semidiameter of the earth r, and the distance of the body d,

$$r = d . \sin . \mathrm{H}.$$

If, then, we know the radius of the earth and the horizontal parallax, we can find the distance of the body, or knowing the distance, we can find the horizontal parallax.

43. We are not yet in a position to point out here any method by which the moon's distance, and consequently her horizontal parallax, may be determined, chiefly because her proximity to the earth causes the variable radius of the earth to exercise considerable influence over the result in her case; but, in other cases, the following method will be applicable.

44. Supposing the three points O, O′ and M to be in the same

plane as the centre of the earth N, then, calling the zenith distances ZOM, Z′O′M (which are supposed to be simultaneously observed), z and z'; the parallactic angles O′MN, OMN, p' and p; the distance MN, d; the known angle ONO′ between the two stations, l, and the radius of the earth r, we have

$$p=z-\mathrm{MNO} \text{ and } p'=z'-\mathrm{MNO}',$$

$$\therefore \quad p+p'=z+z'-l; \qquad (1.)$$

but in the triangle ONM, we have

$$\sin p=\frac{r}{d}.\sin z; \qquad (2.)$$

and similarly,

$$\sin p'=\frac{r}{d}.\sin z';$$

but p and p' are both very small, they will therefore not differ materially from their sines; we have then

$$p+p'=\frac{r}{d}.(\sin z+\sin z');$$

and, combining this with (1.),

$$d=\frac{r.(\sin z+\sin z')}{z+z'-l}.$$

If we wish to obtain the horizontal parallax, we have only to combine this result with that above given, viz.

$$r=d.\sin \mathrm{H},$$

and we obtain

$$\mathrm{H}=\frac{z+z'-l}{\sin z+\sin z'},$$

since H and its sine are, as before, interchangeable.

The parallax in altitude, that is, the parallax when the star has a zenith distance z, can very readily be obtained from the horizontal parallax, for, by (2.), we have—

$$p=\frac{r}{d}.\sin z; \text{ and } \mathrm{H}=\frac{r}{d} \text{ (art. 42.)},$$

whence

$$p=\mathrm{H}.\sin z.$$

It will be interesting to inspect the following table of parallaxes of the sun and moon at various zenith distances, in order to see how different is its amount in these two cases.

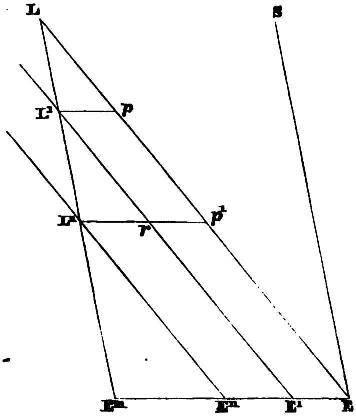
	Parallax of			Parallax of	
Zen. Dist.	Sun.	Moon.	Zen. Dist.	Sun.	Moon.
	″	′ ″		″	′ ″
0	0	0 0	50	7	44 23
10	2	9 54	60	8	50 14
20	3	19 42	70	8	54 34
30	4	28 53	80	9	57 13
40	6	37 12	Hor. par.	9	58 6

The values of the parallax of the moon above given are only true for the particular horizontal parallax with which they terminate, which is itself subject to considerable change, owing to her variable distance and the variation in the length of the earth's radius in different latitudes.

On Aberration.

45. There is yet another apparent displacement of the heavenly bodies arising from the earth's motion, which has been already alluded to (art. 9.), and which, although too small in amount to influence the accuracy we shall be likely to attain in the results of ordinary observations, is very well worth our attending to, as the only incontestable direct proof of the earth's motion in her orbit. This new source of error has its origin in the physical property of light, which, as we have shown, requires that a certain interval elapse between the instant of emission of a ray of light and that of its reception at any distant point. The rapidity of transmission is, however, so great that it is only when we come to apply the principle of instantaneous transmission, which the experience of common life would lead us to adopt, to the immense distances with which we have to deal in our astronomical researches, that its falsity becomes apparent to our senses.

Fig. 9.



46. Let E, E′, E″, E‴ be four successive positions of the earth in her orbit, and let L, L′, L″ be the corresponding places of a ray of light, which arrives at E‴ at the same instant as the earth, proceeding in the direction LE‴, parallel to SE, the course of another ray from the same star: then, if we conceive that we are looking at that star through an immensely long tube, it is clear that the tube must be so inclined as to enclose the ray L at every

point of its course; supposing it therefore placed in the direction EL, we find that, if it be kept always parallel to itself, when the ray L has arrived at L', the point p of the tube will also have arrived at that point, since $LL' : LE''' :: pL' : EE'''$, and therefore it will still enclose that ray; again, when the ray has arrived successively at L'' and at E''', the points p' and E of the tube will also arrive at those positions respectively, and consequently the tube will have enclosed the same ray L in every part of its course. It follows, therefore, that in order to see from the position E the star whose rays proceed in the direction SE, our tube must not be placed in the line ES, but inclined to that line in the direction EL; the amount of deviation, SEL, from the true direction of the star obviously depending upon the relative rapidity with which the particle L and the earth E move in their respective courses, and which determines the ratio of LE''' to EE''' in our triangle. This deviation is called the "*aberration*" of the body observed.

47. All the fixed stars are found, by careful observation, to shift their positions in accordance with the above theory, to an amount dependent upon the direction and velocity of the earth's motion in her assumed orbit, and we may therefore consider that her motion is an ascertained fact.

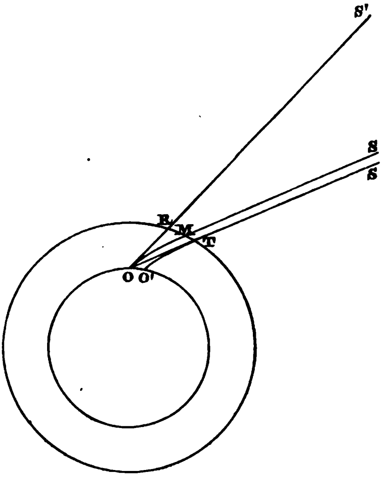
On Refraction.

We have now illustrated all the direct points of connexion between the earth's motion and the appearances of the heavens which will be the subjects of our observation, but there still remains a source of error, which, though dependent, like the last, upon a physical property of light, is in no respect traceable to the earth's motion. The property alluded to is the "*refraction*," or bending aside, which a ray of light undergoes when in its course it passes from one medium into another of a different density.

48. It appears from what has been said in art. (15.), that, when a ray is known to pass from one medium to another, allowance must be made for the refraction which it undergoes, if we would arrive at correct conclusions from any observations we may make upon it after it has suffered this change. Now in passing from any star to the earth, every ray must enter the atmosphere by which this globe is surrounded, and must therefore undergo the continual change of direction due to the continually increasing density of the atmosphere as the ray reaches the lower regions; its course will

then necessarily be slightly curved downwards, as in the figure, and the star which an observer at O would, but for the atmosphere, see in the direction OS, would, owing to the refraction, be seen by him in the direction OS′. For the ray whose course is ST would, by the effect of refraction, be compelled to take the course TO′, and would, therefore, not reach the observer at O; the ray actually observed would be that whose course is SM, which would be curved downwards to O, and the star from which the ray proceeded would be seen in the direction OS′ tangential to the curved course at O. The angle S′OS between the true direction SO and the apparent one S′O is termed the "*refraction*" of the star S.

Fig. 10.

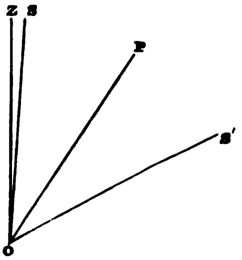


49. The amount of refraction depends upon the zenith distance of the star, as is readily seen, because the greater the zenith distance the broader the portion of air through which the ray must pass before it reaches the eye of the observer; and that amount, whatever it may be, must evidently be *added* to the observed zenith distance of the star in order to eliminate the error arising from this cause.

50. This "correction," which must be applied to every zenith distance observed with the necessary instruments, of which we shall presently speak, is deduced by analytical processes from the known physical constitution of the atmosphere, combined with certain theoretical assumptions concerning some points which still remain comparatively uncertain, such as the law of decrease of temperature of the air as the height above the surface of the earth increases; but the close accordance of the results of theory with those of direct observation is so remarkable, that it is only at zenith distances very near 90°, that any considerable uncertainty can be said still to exist. In order to show how the results of such theoretical investigations can be compared with those derived from direct observation, let O

be the place of observation, OZ the direction of the zenith, OP that of the pole of the heavens, OS the direction in which a star, very near the zenith, is seen when in the vertical plane passing through the zenith and the pole, and OS′ the direction in which the same star is seen when, after the earth has made a half revolution upon her axis, it is again in that plane; then, from what was said in art. (29.), the angle SOP should be equal to the angle S′OP, supposing there were no refraction, and therefore to whatever extent they may be found to differ, that amount must be due to the change which the direction of the star at S′ has suffered from that cause, since, being close to the zenith at S, it there undergoes no appreciable change: now the angles ZOS and ZOS′ can be measured without difficulty, so that these and ZOP being known values, we have

Fig. 11.



$$\text{SOP} = \text{ZOP} - \text{ZOS},$$

and $$\text{S}'\text{OP} = \text{ZOS}' - \text{ZOP};$$

the difference (SOP − S′OP) is, as we have shown, due to the refraction, and may be compared with the results of theory.

51. Table I. at the end of the volume, shows the amount of "*mean*" refraction, that is, the refraction irrespective of the small allowance which must be made on account of the variation of the atmospheric temperature and pressure from certain arbitrary standard values which have been employed in the investigation of these results. Table II. shows the factor by which the values in Table I. must be multiplied when the barometer and thermometer differ from the mean state assumed in Table I.

CHAPTER III.

Illustrative astronomical figure.—Measurement of altitude and azimuth.—Polar distance.—The meridian.—The horizon.—Vertical circles.—The prime vertical.—Meridional observations for geographical latitude.—Examples.—The Ecliptic.—Obliquity of the Ecliptic.—Signs of the zodiac.—Precession.—Right Ascension.—Latitude and longitude of a body.—Measurement of time.—Sidereal time.—Transit of first point of Aries.—Catalogues of stars.—Solar time.—Variable motion of the sun in Right Ascension.—Mean solar time.—Apparent solar time.—Equation of time.—Equation between sidereal time, apparent time, and Right Ascension of the sun.—Equivalent intervals of sidereal and mean solar time.

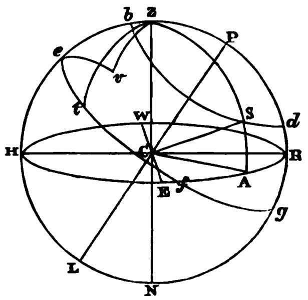
HAVING illustrated the nature of the apparent motions of the stars and pointed out the errors which the peculiarities of our position on the surface of the earth necessarily cause in the results which our mere sense of sight would gather from the appearances set before it, and the modes of correcting these errors, we now proceed to show how the measurements of which we have been treating may be made, and, by the aid of our assumed observations, to establish a mode of fixing the relative positions of the celestial bodies, which we shall afterwards apply to the practical determination of geographical position under certain assumed astronomical data.

52. But, in preparation for this, it will be well to construct some general figure which will show more clearly than we have yet done, the way in which we may represent the appearances of the imaginary celestial sphere as viewed from a point exterior to it, remembering that all the angular measurements with which we have to do are measured at the centre of that sphere, and that the sphere itself has no real existence, but is a mere artifice to enable us to represent certain arcs which are the measures of the angles which we have to use.

53. Let PL be the direction of the earth's axis of rotation; C the earth considered as an evanescent point in comparison of the distances CP, CZ, CS, &c. from it to the points P, Z, S, &c. supposed to be on the celestial sphere, which we imagine to enclose all the heavenly bodies; HAR a great circle whose plane passes through the centre of the earth at C, and is parallel to the plane

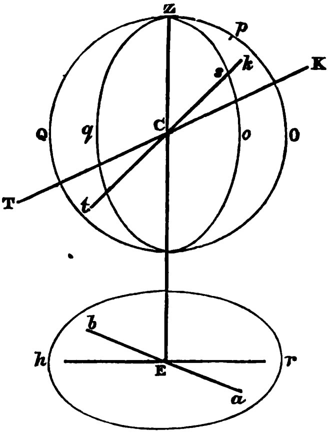
tangential to the surface of the earth at any definite place of observation; CZ a straight line perpendicular to the plane HAR and meeting the sphere in the zenith of that place, Z (art. 25.); HPRL a great circle passing through the points P and Z, and therefore perpendicular to HAR; and ZSA a quadrant of a great circle passing through Z and the point S, which is supposed to be the apparent position of a star seen in the direction CS. Then, if, through S, we draw the small circle *b*S*d* having P as its pole, it is evident that this circle will be the semi-diurnal course which the star S appears to describe about P, owing to the rotation of the earth (art. 30.). R being the point where the vertical circle through P, the celestial pole, cuts the horizontal plane HAR, is the point which we formerly selected as our starting-point in the measurement of azimuths (art. 25.). The angle RCA, or the arc RA, between this point and the plane ZCA, or the angle RZS, is the *azimuth* of S, and the angle ZCS, or the arc ZS, is the *zenith distance* of S, and is complementary to the angle SCA, or the arc SA, which is the *altitude* of the star S. By means of this kind of figure we can readily indicate any particular position of a star, and its construction will always assist in giving a clear conception of the practical problems whose solution we shall presently enter upon.

Fig. 12.



54. In order then to measure simultaneously the two angles RZS and ZCS, which are the *azimuth* and *zenith distance* of S, we must possess an instrument, which, by the combination of two movements, the one horizontal and the other vertical, will enable us to direct the telescope which is attached to it, towards any particular star we may select as the subject of our observation. Let TK indicate the position a telescope fixed to the circumference of a circle ZOQ, the whole having a motion round an axis at C, which

Fig. 13.



is perpendicular to the plane of the circle and therefore to ZE in that plane, so that the telescope may be moved in either direction towards or away from CZ; then, if the line ZE be vertical, the motion of the telescope will be in a vertical plane, and therefore, when the telescope is directed towards a star, the angle ZCK, or the are ZK which is graduated, will measure the zenith distance of that star.

55. Next suppose that the telescope TK, with its attached circle and its axis, have a motion upon the vertical line ZCE, so that it can without disturbing the angle ZCK take up a new position Z*soq*; then if we suppose the line *hr* in the plane ZOQ to be rigidly-attached to the axis ZE, so that it moves round the circle *rah* with this horizontal motion, we see that (E*a* being its new position, when the circle ZOQ is placed in the position *zoq*) the angle *r*E*a*, or the graduated arc *ra*, will measure the arc through which it has been necessary to turn the instrument in order to bring the telescope to bear upon a star in the vertical plane Z*oq*; and if we finally suppose that ZOQ and *hr* were placed in the plane ZCH of the preceding figure, so that EZ were in the same straight line with CZ, and that the telescope were moved horizontally round the axis ZE and vertically in the plane of its attached circle until it pointed in the direction C*s*, corresponding to CS in the last figure, then the graduated arc *ra* would measure the *azimuth*, and the arc Z*s* the *zenith distance* of S.

56. An instrument constructed upon the above principle is called an "*Altitude and Azimuth Instrument,*" and is the most universally useful of all those constructed for a fixed place of observation.

Two outline representations of this instrument are given at the commencement of the volume, having the vertical circle in two distinct positions, at right angles to one another. In those figures F, F, F are the screw feet which enable us to place the axis of the conical spindle at C (upon which the plate HR carrying the supports P, P turns) in a vertical position; the telescope, OE, is attached to the vertical circle G, and moves with it upon the horizontal supports YY, so as, when properly adjusted, to describe by the prolongation of its axis a great circle passing through the zenith. The screws at S and S act upon the supports at YY, so as to enable us to place the axis of motion YY in a horizontal position. The margin of LM in contact with the plate HR is graduated, and the amount of azimuthal motion of the upper part of the instrument is marked by means of an index at V. Similarly, the amount of vertical motion of the telescope is marked by an index at N in contact with the graduated circle G. If this vertical circle be placed in the plane HZPR (fig. 12) it may evidently be employed for the measurement of the distances Zb, Zd; and, since $Pb=Pd$ (when the proper correction for refraction has been applied), we have

$$Pb=PZ+Zb,$$

and

$$Pb=Pd=Zd-PZ,$$

whence, by addition and subtraction,

$$Pb=\frac{1}{2}.(Zb+Zd),$$

and

$$PZ=\frac{1}{2}(Zd-Zb);$$

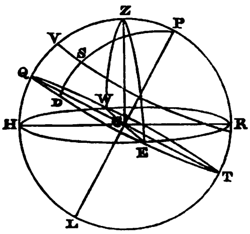
so that, by a simple observation of this kind, both the arc Pb or Pd, which is called the "*Polar Distance*" of the star, and PZ, which is the zenith distance of the pole, can be determined.

57. Every celestial body must, in its daily course, pass the visible portion of the circle RPZHL (fig. 12), namely the portion RPZH, at least once, and the instrument we have been describing may be used to enable us to mark the instant at which any star crosses this circle, which is called the "*Meridian*" of the place of observation, and may be defined as "*that secondary to the horizon which passes through the Pole,*" the circle HAR being called the "*Rational Horizon*" to distinguish it from the "*Sensible Horizon,*" which is supposed to be in a plane tangential to the earth's surface, whereas the plane HAR passes through the earth's centre. It is evident that every heavenly body will be on the meridian when its zenith distance is either the greatest or the least that it has in the course of its apparent diurnal motion round the pole P, for we know that Zb is less, and Zd greater than any arc of a great circle which can be drawn through Z to any other point in the circle dSb, and similarly for the body whose course is gfe, we see that Ze is its least and Zg its greatest zenith distance. Moreover, there must be for every star two points at which its zenith distance is the same, one upon each side of the meridian, and at equal distances from it as measured on the line which marks its course, and where it is consequently seen at equal azimuthal angles from the meridian: for, let t and v be two points in the course of the star which passes the

meridian at *e*, having equal zenith distances *Zt*, *Zv*; then, if P*t* and P*v* were joined, the sides P*t*, *t*Z will be equal to P*v*, *v*Z, and ZP is common, therefore the angle PZ*t*=PZ*v*, and consequently the angle *e*Z*t*=*e*Z*v*. If, then, the angle *t*Z*v* between the two positions of equal zenith distances (or, which is the same thing, equal altitudes) be observed, the half of it, *e*Z*t*, will give the angle through which the instrument must be turned upon its vertical axis, in order to place the plane of its vertical circle in the plane of the meridian.

58. If P (fig. 13) be the *North Pole* of the heavens, then R is the *North* point of the horizon; H the *South* point; E and W at the extremities of the line EW, drawn at right angles to RH, the *East* and *West* points respectively; all the heavenly bodies will therefore rise somewhere in the semicircle REH, and set somewhere in the semicircle HWR, the position of rising or setting being dependent upon the polar distance of the star; in fact, the greater the polar distance, the greater the azimuth of the body at rising. When a body rises exactly at the east point E, it must set at W, if there be no change in its polar distance, and therefore the plane of its diurnal course passes through the point C; it must therefore be the great circle whose pole is P: now the plane of the great circle whose pole is P must necessarily be identical with the plane of the equator of the earth, since both pass through the same point C, and both are perpendicular to the axial line PL: the great circle of the heavens which passes through the east and west points and is perpendicular to PL is therefore called the equator of the celestial sphere. The circles ZA, ZR, Z*v*, Z*t*, &c. are called "*vertical circles*," and that one of them, whose plane is perpendicular to that of the meridian, passes through the east and west points, and is called the "*Prime vertical*"; thus EQWT (fig. 14) is the celestial equator, and WZE the prime vertical of the place whose zenith is Z.

Fig. 14.



59. Since ZC is the direction of that radius of the earth which passes through the place of observation, and EQWT the plane of the earth's equator, it follows that the arc ZQ precisely corresponds to the arc on the surface of the earth, which measures what is called

the "*Latitude*" of the place; if, therefore, we can, by observation, determine the arc ZQ, or, which is the same thing, the angle ZEQ, or the angle ZCQ, we shall have found one of the two values which fix the terrestrial position of a place. Now we have already shown how the arc ZP may be found by the observed meridional zenith distances of a star whose entire course is above the horizon*, and $ZQ=PQ-PZ=90°-PZ$, wherefore ZQ may be found from PZ, which, being its complement, is called the "*co-latitude*" of the place.

60. We have, again, $PQ=ZR$, since both are equal to 90°, wherefore $PR=ZQ$; so that *the altitude of the pole is equal to the latitude of the place of observation.*

61. The distance SD or VQ from the star to the equator measured on a secondary to it, which is the complement of the polar distance PS or PV, is called the "*declination*" of the star S. If the declination of a star be known, the observation of its zenith distance when on the meridian will determine the latitude of the place; for $ZQ=ZV \pm VQ$, according as the declination of the star is on that side of the equator which is towards the elevated pole, or on the other side. This difference of position is indicated by the terms "*North declination,*" when the star is in the northern celestial hemisphere, and "*South declination*" when it is in the southern celestial hemisphere: it is also indicated by prefixing in the former case the sign + to the declination, and in the latter the sign −.

Examples.

1. Given the observed meridian altitude of the sun's centre 56° 31′ 25″, and its declination +16° 1′ 39″; find the latitude of the place of observation.

By Table I. refraction =39″, which being applied to the observed altitude, we get latitude =49° 30′ 53″ N.

2. The meridian altitude of the sun's centre was observed below the pole to be 6° 29′ 41″ when its declination was +22° 53′ 9″; what is the latitude of the place?

Latitude =73° 28′ 36″ N.

3. The meridian altitudes of a circumpolar star are observed to be 7° 10′ 52″ and 71° 16′ 30″; find the latitude.

Latitude =39° 9′ 53″.

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* Such a star is called a "*circumpolar*" star.

4. The meridian altitudes of a circumpolar star in the southern hemisphere were observed to be 19° 32′ 7″ on the southern, and 72° 12′ 4″ on the northern side of the zenith; find the latitude of the place and the declination of the star.

Latitude =63° 38′ 49″ S.
Declination = −45° 51′ 35″

5. The declination of a star is −21° 37′ 15″, and the latitude of the place of observation 51° 28′ 39″ N.; find the meridional zenith distance.

Z. D. =73° 5′ 54″.

6. The meridional altitude of a star, observed on the south side of the zenith, is 57° 18′ 6″, and the latitude of the place 37° 14′ 6″ N.; find the north polar distance of the star.

N. P. D. =85° 28′ 26″.

7. The meridional altitudes of a southern circumpolar star were observed to be 81° 16′ 4″ and 13° 15′ 10″; find its declination and the latitude of the place.

Declination =55° 57′ 35″·5.
Latitude =47° 13′ 30″·5.

8. By an observer situated on the equator of the earth, the meridional altitude of a star was observed to be 27° 13′ 24″. What was the declination of that star?

Declination = ±62° 48′ 29″.

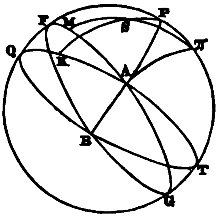
62. If we suppose every point in the imaginary great circle, which we have called the "*equator*," to be fixed by its relative position with reference to certain fixed stars which are adjacent to it, each point will, in succession, pass the meridian of the place of observation, and the interval between two successive transits over the same point of the meridian will be the same as that between two successive transits of any, the same, star.

63. The zenith distance of any fixed star at its "*transit*" across the meridian must, at the same place of observation, remain the same at each successive transit, because we know that its polar distance remains unaltered, except by a very small quantity dependent principally upon aberration. Not so, however, with the sun, the moon and the planets, which, as we have seen, would, owing to the earth's motion in her orbit, in the case of the sun, and that motion combined

with their own proper motion in the cases of the moon and planets, appear to trace out certain paths among the fixed stars. If, then, the zenith distance of the sun be observed from day to day, when that body is on the meridian, it is clear that that zenith distance must be found to be constantly altering, since we should, in fact, be observing the zenith distance of a different point on each successive day. From what was said at art. 36, it appears, that, supposing the sun on any day to have been observed in the direction of some particular star, it must again be observed in the direction of the same star when, at the end of a year, the earth arrives at the same point in her orbit; its declination, therefore, on these two occasions, must be the same, and it must, in appearance, have passed successively all those stars which lie in, or nearly in, the plane of its apparent motion, that is, the plane of the earth's orbit, which plane must necessarily trace out, in its intersection with the celestial sphere, a great circle, since it must pass through the centre of the earth, which is also the centre of the celestial sphere.

This great circle, in which the sun is always to be found, is called the "*ecliptic;*" it intersects the equator in two opposite points (since both are great circles), and the inclination of its plane to that of the equator is evidently measured by the greatest amount of declination which the sun is observed to have, either north or south; thus, if QBTA (fig. 15) be the equator, and FBGA the ecliptic, then the sun will have its greatest declination when at the points F and G; and the declinations FQ and GT measure the inclination of the plane FBGA to the plane QBGT. This inclination is called the "*obliquity of the ecliptic,*" and its measure at the present time is 23° 27′ 32″.

Fig. 15.



64. The ecliptic is divided into twelve equal parts called "*signs,*" so that each sign is equivalent to 30° of arc; and each sign has a particular name attached to it; these names, commencing at the point A (which is supposed to be on the further side of the sphere), and reading the signs in the direction of the sun's apparent motion from A towards F (P being the North Pole) are "*Aries;*" "*Taurus;*" "*Gemini;*" "*Cancer;*" "*Leo;*" "*Virgo;*" "*Libra;*" "*Scorpio;*" "*Sagittarius;*" "*Capricornus;*" "*Aquarius;*" "*Pisces.*" These names were originally given when the point A was in the

constellation or group of stars called now, as formerly, "*Aries*," and so also of the other signs; but, owing to a cause, which we shall presently mention, the point A is continually, though slowly, altering its position, so that it is now to be found considerably distant from the place it then had. The signs, then, retain their names, though they are no longer connected with the constellations to which they owe their titles.

65. The cause of the phenomenon which we have just alluded to, is a slow motion which the pole of the heavens, that is the pole of the equator, has around the imaginary point which is the pole of the ecliptic; this motion is produced by the attractive force which the sun, moon and planets exercise upon the protuberance at the earth's equator (art. 2.), which has the effect of changing the direction in which the earth's axis of rotation points; and from this there necessarily results a corresponding change in the position of the two points of intersection of the equator and ecliptic. The motion of these points, which are called the "*equinoxes* *," is, as we have said, extremely slow, being at the rate of about $50''\cdot1$ annually; but this, in the lapse of years, is quite sufficient to produce a large amount of error in the position of any heavenly body which is referred to those points.

66. One of these points, viz. the point A, where the sun passes from the southern to the northern hemisphere, is called "*the vernal equinox*" or "*the first point of Aries*," although, as before shown, it is no longer in the constellation of that name, but in that of "*Pisces*." And as, by this motion, it, as well as the opposite point, continually *precedes* (in the order in which we read the "*signs*") the position which it occupied at any former epoch, the phenomenon has been styled the "*precession of the equinoxes*."

67. The points of the ecliptic which are severally 90° distant from the equinoxes, are called the "*solstitial points*," or simply "*the solstices*," because, when the sun occupies those positions, its course being parallel to the equator, its meridional altitude from day to day does not materially alter.

68. The principle which we made use of to fix the position of a

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* Because when the sun is at these points in the ecliptic the natural day and night are of equal duration (art. 58.). That point at which the sun crosses the equator from the southern hemisphere to the northern is called, as stated in the text, the "*vernal equinox*," the other is called the "*autumnal equinox*."

star by its altitude and azimuth, will evidently apply to any case of reference to a great circle, whose position we know, and upon which we can determine a point to be considered as an origin from whence to measure the arcs of that circle contained between the origin and the secondaries drawn through the bodies whose positions we wish to fix. If, in fact, we conceive the plane of the horizon to be *tilted up* so as to coincide with the plane of the equator, and turned in its own plane until the point R (fig. 14) coincides with A, (fig. 15), and the point A to be taken as our origin, our measures of azimuth and altitude become transferred, the one to the arc AK (fig. 15) between the origin and the secondary PK drawn through the body S, and the other to the declination KS measured on that secondary.

69. The advantage of this new mode of fixing the point or star S, is that these measures remain unaffected by the diurnal motion of the earth, inasmuch as the origin A and the pole P are unaltered by that motion; whereas the measures of altitude and azimuth determine the position of a star at some particular instant of the day, and for that instant only. We have thus then a means of recording the positions of the heavenly bodies among one another, and with reference to the equator and its pole, which will enable us to arrange them in catalogues or map them upon the surface of a sphere, subject, however, to the small change in the position of the point A arising from precession, and to our ability to determine the measure AK. We have already shown how the measure KS may be found, and we shall presently see how AK may likewise be measured.

70. The arc AK is called the "*right-ascension*" of the body S, and is always measured in the direction of the signs, from 0° to 360°. It is usually characterized by the symbol Æ or A. R., the initials of "*Ascensio Recta.*"

71. The same principle of measurement is adapted to the ecliptic AFBG (fig. 15), by drawing through its pole π the secondary πSM; then the arcs AM and MS will again fix the position of the star S independently of the daily motion. The arc AM is called the "*longitude,*" and the arc MS the "*latitude*" of S. Care must be taken not to confuse these measures with terrestrial latitude and longitude, with which they have nothing in common; these last are indeed analogous to celestial "Right Ascension" and "Decli-

nation," having reference to the same plane of comparison, viz. the plane of the earth's equator, though the origin of the equatorial arcs is different in the two cases; the celestial origin being the first point of Aries, and the terrestrial being so entirely arbitrary that it is not even the same for all the nations of the earth, each having assumed its own point of reference.

72. If the obliquity of the ecliptic be known, the longitude and latitude of a body may without difficulty be determined from its right ascension and declination; for if PA and πA be arcs of great circles, these are both equal to 90°, and therefore A is the pole of πP, so that each of the angles πPA, PπA is a right angle. Now the angle APK is measured by the arc AK, and is therefore equal to the Æ of S; PS is the complement of the declination, or it is the polar distance of S; similarly, AπM is equal to the longitude of S; πS the colatitude of S, and $\pi P = FQ =$ the "obliquity." We have, then, in the triangle πPS—

$$\cos \pi S = \cos \pi P . \cos PS + \sin \pi P . \sin PS . \cos \pi PS, \quad \text{(1.)}$$

$$\cos A\pi S = \sin S\pi P = \sin \pi PS . \sin PS . \text{cosec}\, \pi S; \quad \text{(2.)}$$

or, if the obliquity $=\omega$, the declination $=\Delta$, the latitude $=l$, and the longitude $=L$,

from (1.) $\sin l = \cos \omega . \sin \Delta - \sin \omega . \cos \Delta . \sin Æ$, . (3.)

and from (2.) $\cos L = \cos Æ . \cos \Delta . \sec l$. . . . . . . (4.)

In a similar manner we obtain the corresponding formulæ

$$\sin \Delta = \cos \omega . \sin l + \sin \omega . \cos l . \sin L, \quad \text{(5.)}$$

$$\cos Æ = \cos L . \cos l \sec \Delta, \quad \text{(6.)}$$

the last of which may, however, be at once obtained from (4.).

If, in (3.), we assume $\tan \theta = \tan \omega . \sin Æ$, we get

$$\sin l = \cos \omega . (\sin \Delta - \tan \theta . \cos \Delta)$$

$$= \cos \omega . \frac{\sin (\Delta - \theta)}{\cos \theta} \quad \text{(7.)}$$

And similarly in (5.), putting $\tan \varphi = \tan \omega . \sin L$, we get

$$\sin \Delta = \cos \omega . \frac{\sin (l + \varphi)}{\cos \varphi} \quad \text{(8.)}$$

The formulæ (7.) and (8.) are better adapted to logarithmic computation than (3.) and (5.).

A few examples will illustrate the use of these formulæ.

1. At noon, January 18, 1853, the Æ of the Moon was 40° 54′ 24″·75, and her declination 11° 53′ 49″·7 N.; find her longitude and latitude, the obliquity of the ecliptic being 23° 27′ 31″.

ω = 23° 27′ 31″ . . . tan 9·6374434 . . . . cos 9·9625341
Æ = 40 54 24·75 . . sin 9·8161296 . . . . cos 9·8783924
∴ θ = 15 51 48·1 tan 9·4535730 . . . . sec 0·0168627
Δ = 11 53 49·7 . . . . cos 9·9905694
$\Delta-\theta$ = − 3 57 58·4 . . . . . . sin (neg.) 8·8399074
∴ l = − 3 46 55·8 . . . . . . sin (neg.) 8·8193042 sec 0·0009469
∴ L = 42° 10′ 12″·5 . . . . cos 9·8699087

2. At noon, February 9, 1853, the Æ of the Moon was 339° 47′ 7″·65, and her declination 13° 39′ 9″ S.; find her longitude and latitude, with the same obliquity.

L = 336° 12′ 39″·7
l = 4° 45′ 10″ S.

3. At midnight, March 12, 1853, the Æ of the Moon was 27° 39′ 19″·2, and her declination 6° 56′ 59″·7 N.; find her longitude and latitude, the obliquity being 23° 27′ 32″.

L = 28° 10′ 7″·1
l = 4° 9′ 10″·1 S.

4. At midnight, February 6, 1853, the longitude of the Moon was 302° 46′ 29″·3, and her latitude was 2° 56′ 44″·8 S.; find her Æ and declination, the obliquity being 23° 27′ 31″.

Æ = 305° 47′ 28″·65
Dec. = 22° 25′ 17″.

73. But we have not yet shown how the Æs of bodies may be determined. We have, however, pointed out one method by which the plane of the vertical circle in our altitude and azimuth instrument may be placed in the meridian of the place of observation; and by now assuming that it has been so placed, and that we are, therefore, able to note the instant at which any body passes the meridian, or in other words, *observe its transit*, we shall be able to illustrate the principles upon which Æs are made subject to observation. In this position it answers the purpose of another instrument, called "*the Transit instrument*," which is, in principle, the same as this, but without any provision for more than a very small amount of motion in azimuth.

If we then wait a certain length of time, we shall observe the same body pass the meridian at or near the same point, and again waiting, it will again pass the meridian. Now the question arises, "Are these successive intervals of time of the same length, or are they not?" And this question can only be fully decided by the additional aid of some instrument which will accurately mark equal small intervals of time, so as to enable us, by counting the number of them between two successive transits, to determine whether there be or be not a change of interval as the body comes, time after time, to the meridian.

74. The foundation of our mode of measuring time is the physical fact, that all simple pendulums of equal length oscillate to and fro in equal intervals of time; if the length be increased the interval marked by each oscillation is increased likewise, and *vice versâ*, according to a definite rule or law which we need not here trouble ourselves with, as the simple fact which we first stated is sufficient for our purpose.

It would be most inconvenient if we were, in making use of our pendulum, obliged actually to count the number of its oscillations between the occurrence of every two phenomena, the interval between which we wish to compare with some other similarly observed, and it is simply for the purpose of making the pendulum record for itself the number of its oscillations or "*beats*," that the system of wheels and pinions, and the artifice of the hands is introduced into our clocks and watches. In the case of "chronometers" (which are, in fact, only a superior kind of watches), the equality of intervals is obtained by means of the alternate partial coiling and uncoiling of a fine spiral spring, which is found to produce that result.

75. Now it is observed that if, by such means, the consecutive intervals between the successive transits of the same fixed star be measured, those intervals are the same, and therefore, if each interval be called a sidereal day, and divided into 24 sidereal hours, each hour into 60 minutes, and each minute into 60 seconds, then, if the length of our pendulum be such that it makes 86400 beats in that interval, the interval between two consecutive beats will be one sidereal second of time.

76. If we could set a clock which marks "*sidereal time*," one, that is, which marks an interval of 24 sidereal hours between the transit

of any fixed star, and the immediately succeeding transit of the same star, at $0^h\ 0^m\ 0^s$ when the first point of Aries is known to be on the meridian of the place of observation, the number of hours, minutes and seconds, which that clock marks when any particular star is observed on the meridian, would be a correct measure of the portion of the equator which has passed the meridian since the first point of Aries; or which is the same thing, a measure of the arc of the equator between the first point of Aries, and the secondary which passes through the star, and therefore, also, a measure of the Æ of the star. If, for instance, the clock marked a hours, b minutes, c seconds, at the transit of the star, then

$$\frac{1}{24}\left(a+\frac{b}{60}+\frac{c}{60^2}\right)$$

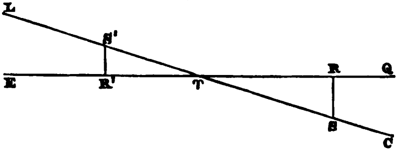
would be the fraction of the equator which measured the Æ of that star, and, if the Æ in degrees, minutes and seconds were required, since 360° is the measure of the whole equator, we have

$$Æ=\frac{360°}{24}\cdot\left(a+\frac{b}{60}+\frac{c}{60^2}\right)=15°\cdot\left(a+\frac{b}{60}+\frac{c}{60^2}\right).$$

If, therefore, the number of hours, minutes and seconds, which, under such circumstances, the sidereal clock marks at the transit of a star, be multiplied by 15, we at once obtain the Æ of that star in degrees, minutes and seconds: and it is plain, that if, conversely, the number of degrees, minutes and seconds in the Æ of any body be divided by 15, we shall obtain the Æ of that body in sidereal hours, minutes and seconds, that is, the sidereal interval which has elapsed, or will elapse, between the transit of the first point of Aries and the transit of the body.

77. The only difficulty then in the determination of the right-ascensions of bodies, is the finding the instant of transit of the first point of Aries: this may be done in the following way. We have seen that we can, without difficulty, regulate our clock to mark sidereal intervals of time: if, then, we observe the meridian altitude of the sun on two successive days, the one before, and the other after he has passed the equinox, and at the same instant we observe the indications of our clock, we shall be able, by a simple proportion (which assumes the motion of the sun to have been equable during the interval), to determine what was the indication of the clock when

Fig. 17.



the first point of Aries was on the meridian. Thus if S and S′ be the positions of the sun in the ecliptic LC at the two transits, and if SR, S′R′ be the declinations, EQ being the equator, we have by similar triangles,

$$\Upsilon R : \Upsilon R' : : SR : S'R',$$

or calling the declinations δ and δ'; the distance R♈, c, and ♈R′, r:—

$$c : r : : \delta : \delta',$$

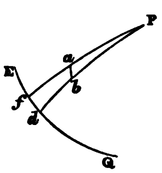
$$\text{or} \qquad c : r+c : : \delta : \delta+\delta',$$

$$\text{whence} \qquad c = \frac{r+c}{\delta+\delta'} . \delta .$$

Now the value $r+c$ or RR′ is the difference of the indications of the clock at the successive transits, and c or R♈ is the sidereal interval which must be added to its first indication, in order to find the indication when the point ♈ is on the meridian; we shall, therefore, by this formula, be able to determine this last, which will of course be the error of the clock from correct sidereal time.

78. It is in such a manner that the ÆRs of all celestial bodies may be obtained; and, having previously shown how their declinations may be found, we now see that the observations of a few successive nights will enable us not only to determine whether any celestial body to which our attention may be directed, holds a fixed position with reference to our plane of comparison (the plane of the equator), or has an apparent motion of its own among the fixed stars, but also the precise amount and direction of that motion; for let EQ be a portion of the equator, P its pole, a and b two successive positions of the body; then aP and bP will be the complements of its declinations, or its polar distances in those positions; and aPb measured by fd, the difference of its ÆRs, so that, having in the triangle aPb, aP, bP, and the angle aPb, we can find the distance ab, and the angle abP, which determines the direction of the motion.

Fig. 18.



79. For practical purposes, one of which we have already adverted to (art. 61.), the fixed stars are arranged in tables, in the order of

their Æs, with their Æs and declinations for certain epochs affixed; but as, owing to the precession of the equinoxes, aberration, &c., these values are continually altering, though to a very small amount, it is necessary, where great accuracy is required, to make allowance for these changes by interpolating the required values between those which are given. As far, however, as our present purpose is concerned, the places of some of the principal stars for the commencement of the year 1853 will be sufficient, and these are given in Table III. at the end of the volume.

80. But any table of this kind is, practically, almost useless, unless we are able to recognize in the sky the particular star named in the table; and the ability to do this can only be acquired by the constant habit of studying on the globe and in the heavens, first, the relative positions of the more remarkable stars, and then, similarly, the situations with respect to these, of other smaller ones grouped among them. To facilitate this kind of reference, the device of grouping the stars in "*Constellations*" was invented in very remote ages. We, in modern times, still adhere to this nomenclature, retaining the Latin names of the constellations, but the Greek numeration of the stars in each, generally, but not always strictly in the order of their relative magnitudes; in the cases, however, of some of the very brightest of them, we also employ traditionary names which have been handed down to us from ages more or less remote. It is thus that we get such names as "Dubhe," *alias* "α Ursæ Majoris;" "Markab," *alias* "α Pegasi;" "Achernar," *alias* "α Eridani;" "Algenib," *alias* "γ Pegasi;" and many others: but it is by no means necessary to burden the memory with this twofold tax; the constellation it belongs to, and the Greek letter which characterizes it, are at once the simplest and most effective means of attaining a knowledge of the "whereabouts" of any star; while for its exact position, nothing short of its Æ and declination, or its latitude and longitude will serve our purpose.

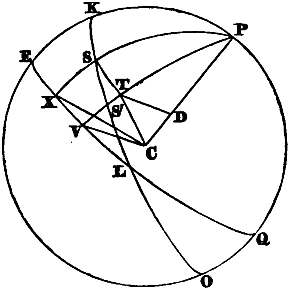
81. We have shown that the sun traces out the circle, which we call the "ecliptic," in its apparent annual course; now this apparent motion is in the contrary direction to that in which the apparent diurnal motion of all the visible celestial bodies proceeds, so that the sun is, from day to day, more and more to the eastward, or less and less to the westward of any particular fixed star, with which we choose to compare his position; and consequently his Æ

must be continually increasing; so that, observed with our sidereal clock, we shall find that the interval between two successive transits of the sun is *greater* than 24 sidereal hours. If we, for a moment, assume that the excess of this interval, which we call a "*solar day,*" over 24 sidereal hours, or a "*sidereal day,*" be always the same, we shall, without difficulty, arrive at its precise amount; for it is evident, that the number of apparent diurnal revolutions made by the sun in the interval between his having the same AR as any particular star, and his having again the same AR (which interval is called *a sidereal year*), must be exactly one less than the number of sidereal days in the same interval.

82. Now the number of sidereal days in a sidereal year is found to be 366·2564, and therefore the number of solar days must be 365·2564; since, then, the sun in the course of 365·2564 solar days loses, as it were, 24 sidereal hours upon the star, it follows that, in one solar day, he will lose $\frac{1}{365\cdot2564} = \cdot002738$ of a sidereal day; this decimal will be found to be equal to $0^{h}\ 3^{m}\ 56^{s}\cdot55$. It appears then that if the rate at which the AR of the sun increases from day to day were considered uniform, $3^{m}\ 56^{s}\cdot55$ would be the sidereal interval by which the solar day would exceed in length the sidereal day; and the solar day would therefore consist of $24^{h}\ 3^{m}\ 56^{s}\cdot55$ in sidereal measure.

83. But there are two causes which interfere with this assumed uniformity of the sun's "motion in AR" (that is, his motion measured on an arc of the equator): one of these is the obliquity of the ecliptic, which causes the direction of his motion in the ecliptic to be less and less inclined to the equator, as he moves from either equinox to the corresponding solstice, while the arc of the ecliptic, which measures that motion, being more and more removed from the equator, is, when transferred to the equator for conversion into change of AR, more and more increased. Thus, in fig. 19, KLO being the ecliptic, ELQ the equator, P the pole, and C the centre of the celestial sphere, let S and S′ be two positions of the sun, very close to one another, then XV is the change of AR

Fig. 19.



corresponding to SS′ the change of longitude: and if we draw the small circle arc ST parallel to the equator; TD perpendicular to the axis PC; and join VC, XC, TC, we have, since TD is the radius of the arc ST,

$$\mathrm{VX} = \mathrm{ST}.\frac{\mathrm{VC}}{\mathrm{TD}} = \mathrm{ST}.\frac{\mathrm{TC}}{\mathrm{TD}} = \mathrm{ST}.\sec.\mathrm{CTD}$$

$$= \mathrm{ST}.\sec \mathrm{TCV} = \mathrm{ST}.\sec \Delta,$$

putting the declination at $S = \Delta$; but STS′ being a very small portion of the sphere, may be treated as a plane triangle, and we have

$$\mathrm{ST} = \mathrm{SS}'.\cos \mathrm{S'ST} = \mathrm{SS}'.\cos i,$$

where i is the plane angle which the sun's course at S makes with a plane drawn through that point parallel to the plane of the equator, whence

$$\mathrm{VX} = \mathrm{SS}' \sec \Delta.\cos i.$$

Now it is seen that, at the equinox L, the angle i (S′ST) is equal to the obliquity, and $\Delta = 0$; while, at the solstice K, $i = 0$, and $\Delta = \omega$ (the obliquity), so that, in the former position, we have

Change of Æ = change of longitude × $\cos \omega$; . . . (1.)

and in the latter,

Change of Æ = change of longitude × $\sec \omega$ . . . . (2.)

And the factor $\cos \omega$ in (1.) being the reciprocal of $\sec \omega$ in (2.), the diminution of the change of longitude, when converted to change of Æ in the one case, is compensated by its increase in the other: we see, moreover, that if the motion in the ecliptic be uniform, the quadrant KL must be passed over in one fourth part of the sidereal year, and therefore the change of Æ, EL, must be made in the same time; so that, at the equinoxes and the solstices, the error arising from our hypothesis, that solar days are all of equal length, would vanish altogether; that is to say, if we had a clock which marked 24 hours in the sidereal interval $24^{\mathrm{h}}\ 3^{\mathrm{m}}\ 56^{\mathrm{s}}\!\cdot\!55$, and that clock were made to indicate $0^{\mathrm{h}}\ 0^{\mathrm{m}}\ 0^{\mathrm{s}}$ when the sun is on the meridian at the vernal equinox, then it would not again mark $0^{\mathrm{h}}\ 0^{\mathrm{m}}\ 0^{\mathrm{s}}$, when the sun is on the meridian, until the summer solstice; after which it would again vary until the autumnal equinox, and so on, throughout the year. The difference between $0^{\mathrm{h}}\ 0^{\mathrm{m}}\ 0^{\mathrm{s}}$ and the time marked by the clock (regulated as above) when the sun is on the meridian, is called the "*equation of time*," and is, in fact, the difference between the "*apparent solar time*" marked by the sun, and "*mean solar time*" marked by the clock.

84. From what has been said it would follow, that if, starting the clock and sun together at the vernal equinox, we could mark out upon the equator the points which, on successive days, are upon the meridian when the clock marks $0^h\ 0^m\ 0^s$, these points would be found to be in advance of the sun (*as regards his motion in the ecliptic*) between the equinox and the solstice, and behind his position from the solstice to the equinox; and the consequence would be that, between the equinox and the solstice the sun would come to the meridian *before* the clock marked $0^h\ 0^m\ 0^s$, and between the solstice and the equinox he would come to the meridian *after* the clock had marked $0^h\ 0^m\ 0^s$; or, which amounts to the same, in order to obtain "apparent time" from the indication of the clock which marks "mean time," we should, on the above hypothesis, have to *add* the equation of time when the sun is passing from an equinox to the corresponding solstice, and to *subtract* the equation of time when he is passing from a solstice to the corresponding equinox.

85. But we have stated that there is still another cause which produces an irregularity of the sun's motion in Æ; this is, the irregularity of its motion in the ecliptic, chiefly arising from the ellipticity of the earth's orbit, which causes the apparent motion of the sun in the ecliptic to be quicker as the distance between the two bodies diminishes, and *vice versâ*.

86. Now the earth is nearest to the sun about the 1st of January in the present year (1853), and therefore, the clock and the sun being together on the previous 24th of December, the "*apparent time*" indicated by the sun will, on two grounds, lose upon the clock; first, because the sun has just passed the solstice; and, secondly, because its apparent motion in longitude is faster than at any other time of the year: moreover, the interval between January 1 and the vernal equinox (March 21) is less than one-fourth of the year. We should therefore expect, as is really the case, that the clock will, before the vernal equinox, have gained so much upon the apparent time, that a considerable interval must elapse after the equinox before the two causes of retardation of the sun's motion in Æ (the obliquity of the ecliptic and the diminished rate of motion in longitude) will counteract this gain and bring the two again into accordance. We find, in fact, that the equation of time at the equinox is as much as $-7^m\ 19^s$, and that it does not vanish altogether before April 15; after which it becomes additive to the mean time marked by the

clock. So it remains until June 14, when it becomes again subtractive; it changes again on August 31, and remains additive until December 24.

87. In all cases in which we record the equation of time, we shall give it the sign + when additive to the mean time; and when subtractive the sign —.

88. The equation of time is tabulated in every astronomical almanac, so that there can never be any difficulty in converting "mean time" into "apparent time:" but, as regards "sidereal time," the case is rather more complicated. The "sidereal time" at any instant is, as we have pointed out, the number of *sidereal* hours, minutes and seconds which have at that instant elapsed since the "first point of Aries" passed the meridian, or, in other words, it is the Æ of the meridian; if, therefore, to the Æ of the sun, in time, there be added the interval of time which has elapsed since "*apparent noon*" (that is, the instant when the sun is on the meridian), the distance, in time, of the first point of Aries from the meridian will be obtained, which is, in fact, the sidereal time at that instant. The Æ of the sun is given usually at apparent noon or at "mean noon" (when our mean time clock marks $0^h\ 0^m\ 0^s$); if, therefore, the sidereal time at any other instant be required, it will be necessary to interpolate the precise value of the Æ at that instant, which, owing to the small change which takes place from day to day, can very readily be done. The same will hold good of all the tabulated values in any well-arranged almanac, except in cases where very great accuracy is required.

89. If, then, α be the apparent time since the preceding apparent noon, σ the sidereal time at the instant when the sun's Æ is ρ, we have, as the necessary relation between these quantities, $\sigma = \alpha + \rho$.

90. The terms "mean solar time" and "sidereal time," which we have been using, refer to the simultaneous records of the same moment as indicated upon two clocks regulated as has been explained, and these records are connected by the above equation. But when we come to measure a definite interval between one abstract moment and another by means of these clocks, the case is somewhat different; for in determining the connexion which must subsist between the two measures, we must consider the relative values of the same unit of time as estimated by each, because the length of the interval, as indicated by the clock, will depend upon

the abstract length of the unit of measure. Since, then, we know that (art. 82.)

$$\frac{\text{a sidereal year, as a sidereal interval}}{\text{a sidereal year, as a mean solar interval}}=\frac{366{\cdot}2564}{365{\cdot}2564}=1{\cdot}002738,$$

$\therefore$ any sidereal interval $=1{\cdot}002738\times$ corresponding mean solar interval. Similarly,

$$\text{any mean solar interval}=\frac{1}{1{\cdot}002738}\times\text{corresponding sidereal interval}$$

$$=0{\cdot}99727\ \times\text{corresponding sidereal interval.}$$

Hence, in order to convert a mean solar interval into the corresponding sidereal interval, we must add to it, for each hour, ·002738 of an hour, or 9·856 seconds; and in order to convert a sidereal interval into the corresponding mean solar interval, we must subtract ·00273 of an hour, or 9·83 seconds for each hour in the sidereal interval.

Examples.

1. The equation of time being $+14^{m}\ 53^{s}$, and the Æ of the sun $15^{h}\ 27^{m}\ 34^{s}$; find the sidereal time corresponding to $7^{h}\ 33^{m}\ 12^{s}$ mean time.

Sidereal time $=23^{h}\ 15^{m}\ 39^{s}$.

2. The mean time being $21^{h}\ 13^{m}\ 27^{s}$, the equation of time $-9^{m}\ 47^{s}$, and the sidereal time $16^{h}\ 53^{m}\ 21^{s}$; find the Æ of the sun.

Æ of sun $=19^{h}\ 49^{m}\ 41^{s}$.

3. The Æ of the sun being $5^{h}\ 14^{m}\ 2^{s}$, the sidereal time $15^{h}\ 37^{m}\ 10^{s}$, and the equation of time $+12^{m}\ 53^{s}$; find the mean time.

Mean time $=10^{\circ}\ 10'\ 15''$.

4. Find the sidereal time of sunset at the autumnal equinox, and thence the sidereal time at the following mean noon; the equation of time being $+7^{m}\ 20^{s}$ at sunset.

Æ of sun at mean noon = Sidereal time $=12^{h}\ 10^{m}\ 18^{s}{\cdot}6$.

5. Find the equation of time when the sidereal clock marks $15^{h}\ 10^{m}\ 4^{s}$, and the mean time clock $4^{h}\ 7^{m}\ 23^{s}$, the Æ of the sun being $10^{h}\ 58^{m}\ 4^{s}$ at that instant.

Equation of time $=+4^{m}\ 37^{s}$.

6. The sidereal time of sunrise being $20^{h}\ 14^{m}\ 37^{s}$, and the Æ of the sun $3^{h}\ 5^{m}\ 53^{s}$; find the equation of time at the following apparent noon, supposing the mean time of sunrise to be $17^{h}\ 0^{m}\ 0^{s}$, and that the equation of time at the previous noon was $+8^{m}\ 20^{s}$.

Equation of time $=+8^{m}\ 53^{s}{\cdot}6$.

7. Find the mean solar interval corresponding to the sidereal interval $8^h\ 43^m\ 27^s$.

Mean solar interval $=8^h\ 42^m\ 1^s{\cdot}2$.

8. Find the sidereal interval corresponding to the mean solar interval $17^h\ 54^m\ 30^s$.

Sidereal interval $=17^h\ 57^m\ 26^s{\cdot}52$.

9. The transit of a star on the equator was observed at $7^h\ 15^m\ 43^s$ mean time; find the mean time of its setting.

Mean time $=13^h\ 14^m\ 44^s{\cdot}02$.

10. The sidereal time at apparent noon was $21^h\ 14^m\ 33^s$; find the sidereal time at $6^h\ 0^m\ 0^s$ apparent time.

Sidereal time $=27^h\ 15^m\ 32^s{\cdot}14$.

11. The mean time of the transit of the first point of Aries was $13^h\ 56^m\ 4^s$; find the mean time of transit of a star whose Æ was $6^h\ 14^m\ 37^s$.

Mean time $=20^h\ 9^m\ 39^s{\cdot}63$.

12. A star whose Æ was $14^h\ 8^m\ 40^s$, was observed at $9^h\ 7^m\ 10^s$ mean time, which corresponded to $17^h\ 4^m\ 5^s$ sidereal time; find the mean time when the star was on the meridian.

Mean time of transit $6^h\ 12^m\ 13^s{\cdot}74$.

13. The sidereal time of transit of the sun's centre was $13^h\ 12^m\ 58^s$, and the sidereal time at mean noon was $12^h\ 58^m\ 41^s$; what was the equation of time?

Equation of time $=14^m\ 14^s{\cdot}66$.

14. The sidereal time at mean noon was $17^h\ 54^m\ 31^s$, and the Æ of a star $3^h\ 7^m\ 54^s$; what was the mean time of its nearest transit?

Mean time of transit $9^h\ 11^m\ 52^s{\cdot}34$.

15. The transit of α Arietis was observed at $8^h\ 14^m\ 34^s$ mean time; what was the mean time of the transit of Rigel the same night?

Mean time of transit of Rigel $11^h\ 22^m\ 37^s{\cdot}11$.

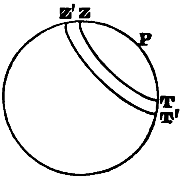
CHAPTER IV.

DETERMINATION OF GEOGRAPHICAL LONGITUDES, ETC.

Determination of Geographical Longitude—by transit observations of a fixed star—by chronometer—by eclipses of Jupiter's satellites—by transit observations of the moon's bright limb.—Examples.—Ex-meridional observations for the determination of Latitude and Longitude.—The triangle formed by the zenith distance, the polar distance and the colatitude.—Recapitulation of trigonometrical formulæ of solution for spherical triangles.—Examples.—Transit observations on the prime vertical.—Equation of equal altitudes.—Examples.—The Sextant.—Lunar distances.—Examples.

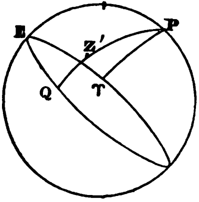
91. We have already shown (art. 56.) how, by the meridional observation of a "*circumpolar*" star, both the polar distance of the star and the latitude of the place of observation may be found; and in art. 61. we showed that when the declination of a star is known, it can at once be made subservient to the determination of the terrestrial latitude. On much the same principle, applied to the Æs of stars, combined with their observed times of transit, may differences of terrestrial longitude be calculated; for just as the difference of the zenith distances of the pole at two places whose zeniths trace out the small circles ZT and Z′T′ (fig. 20), will give the arc ZZ′ which measures their difference of latitude; so the difference of the Æs of the meridians of two places, at the same instant, will give the arc of the equator intercepted by those meridians, which, when transferred to the equator of the earth, is, in fact, the difference of the longitudes of those places.

Fig. 20.



For let ♈EQ (fig. 21.) be the celestial equator, P the north pole of the heavens, PZE the meridian of the place most to the westward, PZ′Q the meridian of the other, and ♈ the first point of Aries; then ♈E is the Æ of one meridian and ♈EQ that of the other, and their difference EQ will evidently correspond to the arc of the terrestrial equator between their terrestrial meridians, and will therefore measure their difference of longitude.

Fig. 21.



Now ♈E, or the angle ♈PE, is the sidereal time at the place whose meridian is PZE and ♈EQ, or the angle ♈PQ is the sidereal time at the place whose meridian is PZ′Q; consequently, "*the difference of the sidereal time at any two places at the same instant, is equivalent to the difference of the terrestrial longitudes of the two places.*"

92. In order to find the sidereal time at our own place of observation, we have only to observe the transit of any star whose Æ is marked in our catalogue; if then we know, or can find by any means, the sidereal time at the other place when our observation was made, we at once obtain the difference of longitude of the two places; and, if either place be on that meridian from which all terrestrial longitudes are measured (the meridian of Greenwich), this difference will be the longitude of the other place, *east,* if the time at that place is later than at Greenwich, and *west,* if the contrary.

93. What has been here said of sidereal time applies equally to mean solar time and to apparent time, only that, in the latter case, we have no direct means of obtaining its exact amount at the distant place, except through the intervention of either of the others, which can always be transferred from place to place by the use of well-adjusted chronometers. Mean time is, however, almost universally adopted for determining the position of the meridian of the distant place, but whatever may be the kind of local time in which the instant of observation may be marked, the two must be reduced to the same standard for the purpose of comparison.

94. We have already remarked that the planet Jupiter is accompanied in his orbit by four satellites, which are distinctly visible through a moderately good telescope, and that the motions of these bodies are so well known that their positions with respect to their primary (Jupiter) can be, with certainty, predicted for a considerable period; the consequence is that the exact time at which any one of them disappears in the shadow of Jupiter or emerges from it can be, and is, predicted, as occurring at a certain instant of Greenwich mean time*. An event of this kind is simultaneously seen from all parts of the earth from which the planet is visible; if, then, at any place, the local mean time at which a particular satellite is eclipsed be observed, a reference to the almanac will give the corresponding mean time at Greenwich, and the difference of these will be the longitude in time, which value, multiplied by 15, will give the longitude in degrees, &c.

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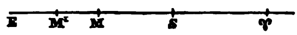
* Table IX.

ON FINDING THE LONGITUDE BY OBSERVED TRANSIT OF THE MOON'S BRIGHT LIMB.

95. The Moon, owing to her proximity to the earth, moves with very considerable angular velocity among the fixed stars, and therefore her distance from any star to the westward of her *increases,* and from any star to the eastward of her *diminishes* very rapidly: consequently her exact distance from any star will denote a particular instant of time, and any change in her distance will indicate the lapse of a definite number of hours, minutes and seconds. One mode of applying this principle to the determination of the distance *in time,* of any particular meridian from that of Greenwich, is to observe the transit of the "*bright limb*" of the moon (that is, the *edge* which, being the visible boundary of that part of her surface which is most directly turned towards the sun, is most sharply defined against the sky), and, either immediately before or immediately afterwards, the transit of some particular star, the difference of whose Æ from that of the moon's bright limb, when on the meridian of Greenwich, is known; then, knowing the change of the moon's Æ in one hour and having by the above observation obtained the increase of her Æ since the time of her transit across the meridian of Greenwich, we readily obtain the time which has elapsed since her transit at Greenwich, which will be the longitude of the place *plus* the change of the moon's Æ.

Thus if E♈ be a portion of the celestial equator, ♈*s* the Æ of the star, ♈M the Æ of the moon when on the meridian of Greenwich, ♈M′ the Æ of the moon when on the distant meridian, which in this case will be to the westward (since the moon's motion being to the eastward, she must have passed the meridian of Greenwich), we should observe the value SM′ and obtain SM from the almanac, and so get the resulting value of MM′, which is the change of Æ the moon undergoes between her passage over the two meridians; this value, divided by the change of Æ for one hour, will give the time which has elapsed since the moon was on the meridian of Greenwich; but inasmuch as, when on that meridian, her Æ was ♈M and it is now ♈M′, we must deduct the difference MM′ from this time in order to obtain the difference of the times of transit of the same point M over the two meridians, which difference is the



longitude of the place of observation. If, then, r be the change of the moon's Æ between the two transits, ϱ the change for one hour, and L the longitude of the place, we have

$$L = \frac{r}{\varrho} - r = \frac{r}{\varrho}(1 - \varrho).$$

It is, however, now usual to give in the Nautical Almanac not only the change of the moon's Æ in one sidereal hour, as we have assumed above, but the change for one hour of longitude, the change, that is, which takes place in her Æ in the interval which elapses between her transits across two meridians equidistant from that of Greenwich, and one hour (or fifteen degrees) distant from one another*: so that, calling this value ϱ', we have simply

$$L = \frac{r}{\varrho'}.$$

Another method of finding the longitude by the measured distance of the moon's bright limb from a star, or from either limb of the sun, will be dwelt upon presently.

Examples.

1. The star Regulus was observed to pass the meridian when a chronometer regulated to Greenwich mean time marked $15^h\ 27^m\ 32^s$; what is the longitude of the place of observation, the equation of time being $+5^m\ 8^s$, and the Æ of the sun $17^h\ 12^m\ 3^s$?

	h	m	s	
Greenwich Mean Time .	15	27	32	
Equation of Time . . .		5	8	
Greenwich Apparent Time	15	32	40	
Æ of the Sun	17	12	3	
Æ of Meridian of Greenwich	8	44	43	= Greenwich Sidereal Time.
Æ of Regulus	10	0	32	= Local Sidereal Time.
Longitude of place . . .	1	15	49	East.
			3	
	3	47	27	15
			5	
Longitude of place . .	18°	57′	15″	East.

---

* Table VIII.

2. At $16^{h}\ 52^{m}\ 13^{s}$, Greenwich mean time, the star Sirius was observed to pass the meridian; the Æ of the sun was $21^{h}\ 14^{m}\ 38^{s}$; the equation of time $-14^{m}\ 18^{s}$: what was the longitude of the place?

	h	m	s	
Greenwich Mean Time .	16	52	13	
Equation of Time . . .		−14	18	
Greenwich Apparent Time	16	37	55	
Æ of the Sun	21	14	38	
∴ Greenwich Sidereal Time .	13	52	33	
Local Sidereal Time . .	6	38	40	(= Æ of *Sirius*).
∴ Longitude of place . . .	7	13	53	West.
			3	
	21	41	39	
			5	
Longitude of place . .	108°	28′	15″	West.

3. At $2^{h}\ 15^{m}\ 21^{s}\cdot 5$, Greenwich mean time, the transit of the star α Lyræ was observed; and it was found that the Æ of the sun was, at that instant, $7^{h}\ 23^{m}\ 16^{s}\cdot 2$, and the equation of time $-5^{m}\ 8^{s}\cdot 6$: what is the longitude of the place at which the observation was made?

Longitude 134° 37′ 13″·5 East.

4. The transit of the sun's centre was observed at $12^{h}\ 13^{m}\ 4^{s}\cdot 2$, Greenwich mean time, when the equation of time was $+10^{m}\ 14^{s}\cdot 7$; find the longitude of the place.

Longitude 185° 49′ 43″·5 West.

5. The transit of β Leonis was observed at $22^{h}\ 17^{m}\ 53^{s}\cdot 7$, Greenwich mean time, when the equation of time was $+3^{m}\ 38^{s}\cdot 8$, and the Æ of the sun $2^{h}\ 57^{m}\ 14^{s}\cdot 1$: what is the longitude?

Longitude 155° 41′ 36″ East.

6. At $18^{h}\ 42^{m}\ 7^{s}$, local mean time, the first satellite of Jupiter was observed to disappear in the shadow of the planet; and, upon reference to the almanac, this phenomenon was found to be predicted as taking place at $11^{h}\ 51^{m}\ 26^{s}$, Greenwich mean time: what was the longitude of the place of observation?

Longitude 102° 40′ 15″ East.

7. The difference of the times of transit of δ Piscium and the moon's bright limb was observed to be $0^{h}\ 27^{m}\ 21^{s}$. At her Green-

wich transit the almanac states that their Æs were δ Piscium $0^h\ 41^m\ 2^s$, Moon's bright limb $1^h\ 4^m\ 56^s{\cdot}33$, and the change of the moon's Æ for one hour of longitude was 114 seconds; what is the longitude of the place, the star being to the westward of the Moon?

Longitude 27° 11′ 36″·33 West.

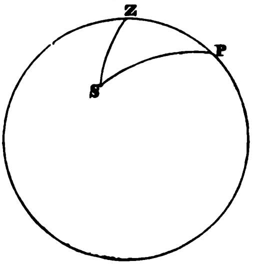
8. The difference of the times of transit of θ Virginis and the moon's bright limb, was observed to be $12^m\ 15^s$; and the Æs at Greenwich transit were, θ Virginis $13^h\ 2^m\ 20^s{\cdot}97$, the Moon's bright limb $13^h\ 32^m\ 5^s{\cdot}39$, and the change of the moon's Æ for one hour of longitude 136·69 seconds: what was the longitude of the place, the star being to the westward?

Longitude 115° 9′ 38″ East.

ON EX-MERIDIONAL OBSERVATIONS FOR THE DETERMINATION OF LATITUDE AND LONGITUDE, ETC.

96. In the methods of determining the terrestrial latitude and longitude which we have hitherto given, we have adapted observations made in the plane of the meridian to that purpose, and these are, in fact, the simplest modes in which such results can be obtained. But one of these values, the latitude, depends entirely upon the zenith distance of the pole ZP (fig. 14); and the other, the longitude, upon the time at the place, as contrasted with the Greenwich time, the former of which times is always known when we have ascertained the magnitude of the angle ZPS at the pole, measuring the angular distance on the equator between the body S and the meridian. Whatever observations, therefore, will enable us to assign at any instant definite values to these two arcs, will suffice to fix the geographical position of the place of observation. Now ZP and the angle ZPS are, under all circumstances, constituent parts of the triangle ZPS formed by the co-latitude ZP, the polar distance PS and the zenith distance ZS of the body at S; consequently, the solution of this triangle in all possible cases will answer our purpose as well, though not quite so readily, as the meridional observations we have before used.

Fig. 22.



The angle ZPS is called the "*hour-angle*" of the body at S, and the angle PZS is the azimuth of S; it appears then that in the sides and angles of this triangle we have the means of establishing the relations between the five quantities, z the zenith distance, δ the co-declination, or polar distance, Z the azimuth, H the hour-angle of S, and λ the co-latitude of the place of observation; any three of which being given, the remaining two can be determined. Of these five quantities, δ, Z and λ can only be known from previous observations, the first being almost always taken from the almanac; the values of the others, z and H, are immediately derivable from observation, but, in all cases, those quantities are selected which are most readily and accurately attainable under the circumstances in which the observer may happen to be placed; with regard to H, however, it must be remembered that the accuracy of its value, as a result of observation, will always be dependent upon the time-piece, which, being more liable to error (except for very moderate intervals) than the instruments with which the values of z or Z are obtained, should, as a general rule, be avoided as a means of finding any of the other quantities, unless the observer have good reason for placing confidence in his clock or chronometer.

97. We have stated that Z can only be known from previous observations, because the point from which it is measured on the horizon must have been previously found, in order to enable us to measure, with the altitude and azimuth instrument, the horizontal angle which is its value at the instant when the altitude is also observed. The angle PSZ we omit altogether from consideration, as it cannot be made the subject of direct measurement, nor can we apply its value, if found, to any practical purpose, other than the determination of one or more of the five remaining parts of the triangle.

98. It must always be borne in mind that the terrestrial hemisphere (northern or southern) in which the place of observation is situated, always fixes the celestial hemisphere in which the zenith of the observer is, and therefore determines whether P, in our figure, is to be considered as the north or the south pole.

99. The fundamental equations

$$\frac{\sin A}{\sin B}=\frac{\sin a}{\sin b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

$$\left.\begin{array}{l}\cos c=\cos a.\cos b+\sin a.\sin b.\cos C \\ \text{or, if } \tan a.\cos C=\tan\theta; \quad \cos c=\cos a.\dfrac{\cos(b-\theta)}{\cos\theta}\end{array}\right\} \quad . \quad (2.)$$

$$\left.\begin{array}{l} \cot A = \dfrac{\cot a . \sin b - \cos C . \cos b}{\sin C} \\ \text{or, if } \tan a . \cos C = \tan \varphi; \qquad \cot A = \dfrac{\cot a . \sin (b - \varphi)}{\sin C . \cos\varphi} \end{array}\right\} \quad . \quad (3.)$$

$$\tan \frac{1}{2}(A+B) = \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} . \cot \frac{C}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

$$\tan \frac{1}{2}(A-B) = \frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} . \cot \frac{C}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.)$$

$$\tan \frac{1}{2}(a+b) = \frac{\cos \frac{1}{2}(A-B)}{\cos \frac{1}{2}(A+B)} . \tan \frac{c}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.)$$

$$\tan \frac{1}{2}(a-b) = \frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)} . \tan \frac{c}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.)$$

and

$$\tan \frac{1}{2} A = \sqrt{\frac{\sin (s-b) . \sin (s-c)}{\sin s . \sin (s-a)}}, \quad . \quad . \quad . \quad . \quad . \quad (8.)$$

in which a, b, c are the sides respectively opposite to the angles A, B, C of a triangle ABC, will enable us to solve all possible cases of the spherical triangle; but as, in our triangle ZPS, the angles Z and H, and the side δ, may have any values not greater than 180°, and the sides λ and z any values not greater than 90°, the simplest cases will be those in which one or other of them is 90°, producing either a right-angled or a quadrantal triangle, to be resolved by the formulæ embodied in Napier's "Rules." We shall thus arrive at particular classes of problems; thus—

1. When $Z = 90^\circ$; the body observed is on the prime vertical.
2. When $H = 90$; the body is six hours from the meridian.
3. When $z = 90$; the body is rising or setting.
4. When $\delta = 90$; the body is on the celestial equator.
5. When $\lambda = 90$; the place of observation is on the terrestrial equator.

100. We will commence our practical illustrations with a few examples of these five cases, and then proceed to others of more general application.

Examples.

1. The star γ Draconis was observed, when due east, to have an apparent altitude of 64° 31′ 10″; find the latitude of the place of observation, and the sidereal time.

Here we have

$$Z=90^\circ;\ \delta\text{[*]}=38^\circ\ 29'\ 31'';\ z\text{[†]}=25^\circ\ 29'\ 18'';$$

and we have, by Napier's Rules,

$$\cos\lambda=\cos\delta\ \sec z\ .\ .\ .\ .\ .\ .\ .\ .\ .\ \ (1.)$$

$$\sin H=\sin z\,.\operatorname{cosec}\delta,\ .\ .\ .\ .\ .\ .\ .\ .\ \ (2.)$$

whence we find $\lambda=29^\circ\ 52'\ 41''$ ∴ Latitude $=60^\circ\ 7'\ 19''$;

	h	m	s
and H =	2	54	57·7
Æ of γ Draconis =	17	53	12·0
∴ Sidereal Time =	14	58	14·3

2. Regulus was observed to be due west at $14^h\ 12^m\ 27^s$, sidereal time; find its altitude at that time, and the latitude of the place.

Here H is the difference of the Æ of Regulus and the observed time; δ is also known from Table III., and hence we find the values:—

Latitude = 26° 21′ 11″ N.; Altitude = 29° 38′ 44″.

3. The Æ of Capella being $5^h\ 5^m\ 50^s$, its altitude was observed at $23^h\ 5^m\ 50^s$ sidereal time, to be 27° 42′ 10″; what was the azimuth of the star, and the latitude of the place?

Here $H=90^\circ;\ z=62^\circ\ 19'\ 41'';\ \delta=44^\circ\ 9'\ 27''$,

and we find,

Azimuth = 51° 52′ 10″ E. of N.; Latitude = 40° 20′ 25″ N.

4. At $18^h\ 0^m\ 0^s$, apparent time, the altitude of the sun's lower limb was observed to be 17° 16′ 39″, and the azimuthal angle of his east (or *left-hand*) limb from a terrestrial object to the south 110° 15′ 27″; and his declination and semidiameter were found in the Almanac to be 20° 32′ 5″ N. and 16′ 12″: what is the latitude of the place, and the azimuth of the object, east or west of south?

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[*] Derived from Table III. [†] Applying the correction for refraction, Table I.

Here the semidiameter of the sun found in the Almanac must be added to the altitude of his lower limb, and, when multiplied by the cosecant of the Z. D.*, subtracted from the azimuthal angle between his east limb and the object, to obtain these values with reference to his centre.

We have then—

Apparent Altitude of sun's lower limb		17° 16′ 39″
Refraction		0 3 6
Parallax	0 0 8″	17 13 33
Semidiameter	16 12″	0 16 20
True Altitude of sun's centre		17 29 53
True Z. D. of sun's centre		72 30 7

Also $\delta = 69° 27' 55''$ and $H = 90$;

whence we find Latitude $= 59° 0' 1''$;

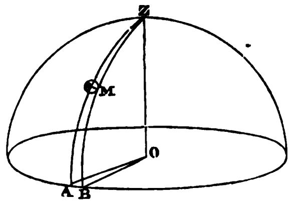
and also azimuth of sun's centre (from N.)	79° 4′ 51″
Semidiameter × cosec Z (subtract)	0 16 58
Azimuthal angle between sun's E. limb and object	110 15 27
Azimuth of object from N.	189 3 20
∴ Azimuth of object W. of S.	9 3 20

5. Find the mean time of the true sunset in latitude 49° 13′ 27″ N. when the sun's declination is $+18° 56' 39''$, and the equation of time $+3^m 54^s$; and find the azimuth of his centre at setting. Find also these values in the same latitude south of the equator.

---

[*] This modification must be made on much the same principle as that observed in art. 83: for let SM represent the semidiameter of the sun; ZA, ZB, two vertical circles through S and M; then, when the angle SMZ is a right angle, the angle Z, or the arc AB, will be the azimuthal value of the semidiameter: hence calling the zenith distance of the centre z, the given semidiameter $MS = s$, and the required angle Z, we have in the right-angled triangle ZSM,

Fig. 23.



$$\sin s = \sin z . \sin Z,$$

whence

$$\sin Z = \sin s . \operatorname{cosec} z;$$

or, since Z and s are always small,

$$Z = s . \operatorname{cosec} z.$$

Calling λ', δ', z', the angles; and Z', H', the sides of the supplemental triangle, we have, in the first case,—

$$\lambda'=139^\circ\ 13'\ 27''; \quad \delta'=108^\circ\ 56'\ 39''; \quad z'=90^\circ;$$

whence we find

$$\left.\begin{array}{l} Z'=119^\circ\ 48'\ 26'' \\ \text{and }\ H'=\ 66^\circ\ 37'\ 10'' \end{array}\right\} \therefore \left\{\begin{array}{l} Z=60^\circ\ 11'\ 34'' \\ H=\ 7^h\ 33^m\ 31^s \text{ Apparent Time.} \end{array}\right.$$

$$\begin{array}{lrrrl} & & 3 & 54 & \text{Equation of Time.} \\ \hline \text{Mean Time of true sunset} & 7 & 29 & 37 & \end{array}$$

In the second case, $\lambda'=139^\circ\ 13'\ 27''$, as before; $\delta'=71^\circ\ 3'\ 21''$; and $z'=90^\circ$; whence

$$\left.\begin{array}{l} Z'=\ 60^\circ\ 11'\ 34'' \\ \text{and }\ H'=113^\circ\ 22'\ 50'' \end{array}\right\} \therefore \left\{\begin{array}{l} Z=119^\circ\ 48'\ 26'' \\ H=\ 4^h\ 26^m\ 29^s \text{ Apparent Time.} \end{array}\right.$$

$$\begin{array}{lrrrl} & & 3 & 54 & \text{Equation of Time.} \\ \hline \text{Mean Time of true sunset} & 4 & 22 & 35 & \end{array}$$

6. The star α Hydræ was observed to have an altitude of $0^\circ\ 34'\ 32''$, at $10^h\ 17^m\ 42^s$ local mean time. Find the latitude of the place; the Æ of the sun being $16^h\ 21^m\ 43^s$, and the equation of time $+6^m\ 11^s$.

The refraction in the horizon being $34'\ 32''$, the star when observed must have been in the horizon; and we have

$$z'=90^\circ; \quad \delta'=81^\circ\ 58'\ 34''; \quad H'=81^\circ\ 18'\ 45'',$$

whence we find, Latitude $=46^\circ\ 54'\ 40''$ South.

7. The altitude and azimuth of the sun's centre before noon when in the first point of Aries, were observed to be Alt. $15^\circ\ 37'\ 10''$, Az. $117^\circ\ 54'\ 22''$; find the latitude of the place of observation, and the sidereal time.

Here $\quad z=74^\circ\ 19'\ 32''; \ \delta=90^\circ; \ Z=117^\circ\ 54'\ 22''$,

whence we find, in the usual way,

Latitude $=60^\circ\ 56'\ 42''$ N. or S.

Sidereal Time $=20^h\ 6^m\ 47^s$.

8. The altitude of Antares, observed, after passing the meridian, at a place on the terrestrial equator, was $48^\circ\ 37'\ 21''$; find its azimuth, and the sidereal time.

Here $\quad \lambda=90^\circ; \ z=41^\circ\ 23'\ 30''; \ \delta=63^\circ\ 53'\ 56''$;

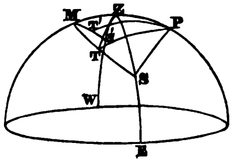
whence we find,

Azimuth $=48^\circ\ 17'\ 16''$ W. of S.

Sidereal time $=18^h\ 33^m\ 46^s$.

TO FIND THE LATITUDE OF A PLACE BY TRANSIT OBSERVATIONS ON THE PRIME VERTICAL.

Fig. 24.



101. If the altitude and azimuth instrument be placed so that the vertical motion be in the plane of the prime vertical, and the two transits of any, the same, star be observed, the one to the eastward at S, fig. 24, and the other to the westward at S′, it is plain that the interval of time between the transits will measure the hour-angle SPS′, and the half of this will be the measure of the angle SPZ. Knowing, therefore, in the right-angled triangle PZS the polar distance PS, and the angle ZPS, we have, for the determination of the latitude,

$$\tan \lambda = \cos H . \tan \delta.$$

This is a very accurate mode of determining the latitude if sufficient care be taken in placing the instrument, and a star be selected which passes the meridian near the zenith; as, in that case, the interval between the transits is too small to be affected with any error which should be allowed to exist in the going of the clock or chronometer with which it is measured. When the star is at S′, it necessarily has the same altitude as at the opposite point of transit S, and by observing this altitude we might dispense with the measurement of the elapsed time between the transits, for in that case, we should have given ZS and SP in the right-angled triangle ZPS, to find ZP the co-latitude.

102. The determination of the latitude by observation of the equal altitudes of a body, is not however restricted to observations on the prime vertical; for, let T and T′ (fig. 24) be two positions of a star when the zenith distances ZT and ZT′ are equal; if these zenith distances be observed, and also the elapsed time, we shall have in the triangle ZTP, ZT the zenith distance, PT the polar distance, and the angle ZPT, half the elapsed time; whence we can find ZP. Or, if instead of observing the elapsed time, the change of azimuth TZT′ be observed, we shall have given in the triangle ZPT, ZT, PT and the angle PZT, the supplement of half the change of azimuth. An observation of this kind is well adapted, under certain circumstances, to the purpose of finding the time of a star's transit over the meridian, when, as at sea, that transit cannot be observed, since

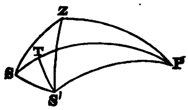
that time is, on account of the equality of the angles ZPT and ZPT′, an arithmetic mean between the times of observed equality of altitudes; but in such a case allowance must be made for the change of place, if any, made by the ship.

103. It has been already shown (art. 57.) that this observation may also be employed for the determination of the position of the meridian, since, for a reason similar to the above, the reading of the horizontal arc of our altitude and azimuth instrument when the vertical circle is in the plane of the meridian, must be an arithmetic mean between the readings when it is in the plane of ZT and in the plane of $ZT_{\prime}$. In all these observations, however, it will be necessary to apply a correction to the results obtained, if, as in the case of the sun and moon, and the planets, the body observed should have undergone a change of declination in the interval between the observations. It is beyond our present purpose to enter upon the investigation of all these corrections; but that which is employed for correcting the time of transit known as

The Equation of Equal Altitudes,

may be thus investigated:—let ZS be the zenith distance of the sun when, *before* noon, the time by chronometer was marked, and let ST be the change of polar distance which takes place between that instant and the instant at which, *after* noon, the sun has the same zenith distance: then constructing the triangle ZS′P, by taking ZS′ and PS′ equal respectively to ZS and PT, we shall have ZPS for the measure of the elapsed time between the first observation and apparent noon, and ZPS′ for the measure of the corresponding elapsed time after noon, and supposing the triangles to be referred to the same side of the meridian, the difference SPS being subtracted from the whole interval between the observations, will evidently give the double of SPZ, which is the interval between the instant of the observation nearest the meridian and the instant of transit.

Fig. 25.



Let $ZPS=H$; $ZPS'=h$; $ZS=ZS'=z$; $PS=\delta$; $PS'=\delta'$;

then in the triangle ZPS we have

$$\cos z = \cos\lambda.\cos\delta + \sin\lambda.\sin\delta\cos H,$$

and, in the triangle ZPS′,

$$\cos z = \cos\lambda.\cos\delta' + \sin\lambda.\sin\delta'.\cos h.$$

Equating these values, we get

$$\cos\lambda.(\cos\delta' - \cos\delta) = \sin\lambda.\{\sin\delta.\cos \mathrm{H} - \sin\delta'.\cos h\},$$

or $$\cot\lambda.(\cos\delta' - \cos\delta) = \sin\delta.\cos \mathrm{H} + \sin(\delta - \delta' - \delta)\cos h$$

$$= \sin\delta.\cos \mathrm{H} + \{\sin(\delta - \delta').\cos\delta - \cos(\delta - \delta').\sin\delta\}\cos h;$$

but, since $\delta - \delta'$ is very small, we may, without sensible error, write $\delta - \delta'$ for $\sin(\delta - \delta')$ and unity for $\cos(\delta - \delta')$; we have then

$$\cot\lambda\,(\cos\delta' - \cos\delta) = \sin\delta.\cos \mathrm{H} + (\delta - \delta').\cos\delta.\cos h - \sin\delta.\cos h$$

$$= \sin\delta.(\cos \mathrm{H} - \cos h) + (\delta - \delta')\cos\delta.\cos h,$$

$$2\cot\lambda.\sin\frac{\delta+\delta'}{2}.\sin\frac{\delta-\delta'}{2} = 2\sin\delta.\sin\frac{h+\mathrm{H}}{2}.\sin\frac{h-\mathrm{H}}{2} + (\delta-\delta')\cos\delta.\cos h,$$

$$(\delta-\delta')\cot\lambda.\sin\frac{\delta+\delta'}{2} = (h-\mathrm{H}).\sin\delta.\sin\frac{h+\mathrm{H}}{2} + (\delta-\delta')\cos\delta.\cos h;$$

$$\therefore \quad h-\mathrm{H} = \frac{\cot\lambda.\sin\frac{\delta+\delta'}{2} - \cos\delta.\cos h}{\sin\delta.\sin\frac{h+\mathrm{H}}{2}}\,(\delta-\delta'),$$

but, since δ and δ', h and H differ by a very small amount, the difference between $\cos h$ and $\cos\frac{h+\mathrm{H}}{2}$ and that between $\sin\delta$ and $\sin\frac{\delta+\delta'}{2}$ are too small to affect our result; hence

$$h-\mathrm{H} = \frac{\cot\lambda - \cot\delta.\cos\frac{h+\mathrm{H}}{2}}{\sin\frac{h+\mathrm{H}}{2}}.(\delta-\delta')$$

or $$h-\mathrm{H} = \left(\cot\lambda.\operatorname{cosec}\frac{h+\mathrm{H}}{2} - \cot\delta.\cot\frac{h+\mathrm{H}}{2}\right)(\delta-\delta').$$

In using this formula the co-latitude λ need be known only approximately; the value of $h+\mathrm{H}$ is the elapsed time between the observations reduced to degrees, and that of $\delta-\delta'$ is the change of polar distance, or of declination during that interval.

Examples.

1. The sidereal interval which elapsed between the transits of γ Draconis over the prime vertical was observed to be $1^{h}\ 17^{m}\ 13^{s}{\cdot}2$; find the latitude of the place, the declination of γ Draconis being $+51^{\circ}\ 30'\ 52''$.

Latitude $=51^{\circ}\ 54'\ 42''$ N.

2. The mean solar interval between the observed equal altitudes of Antares was $4^{h}\ 17^{m}\ 33^{s}$; find the latitude of the place, supposing the observed altitudes to be $54^{\circ}\ 17'\ 27''$.

Reducing the mean solar interval to a sidereal interval, we get $32^{\circ}\ 16'\ 54''$ for the value of H; also $z=35^{\circ}\ 43'\ 14''$, and $\delta=116^{\circ}\ 6'\ 4''$, whence, by using formula (2.), which gives

$$\cos(\lambda \backsim \theta)=\cos z.\cos\theta.\sec\delta, \text{ when } \tan\theta=\tan\delta\cos H,$$

we find, $\theta=120^{\circ}\ 5'\ 31''$, and $\lambda \backsim \varphi=22^{\circ}\ 17'\ 44''$,
which give two values for λ, and from these we obtain—

Latitude $=52^{\circ}\ 23'\ 15''$ S.,
or Latitude $=\ 7^{\circ}\ 47'\ 47''$ S.

3. The mean solar interval between two observed equal altitudes of the sun was $3^{h}\ 27^{m}\ 16^{s}$; the Greenwich mean time of the first observation was $18^{h}\ 7^{m}\ 26^{s}$; the equation of time was $-7^{m}\ 43^{s}$; the declination at first observation was $+10^{\circ}\ 15'\ 7''$, and the approximate latitude $27^{\circ}\ 4'$ N.: find the difference between the time at Greenwich and the time at the place, supposing the diminution of declination in the interval between the observations to have been $2'\ 57''=177''$.

Here $\frac{h+H}{2}=25^{\circ}\ 54'\ 30''$; $\delta=79^{\circ}\ 47'\ 50''$; $\lambda=62^{\circ}\ 56'$,

whence we find $h-H=141''{\cdot}4=9{\cdot}43$ seconds of time; applying this we get—

	h	m	s
Interval from first observation to time of transit	0	43	33·29
Greenwich Apparent Time of first observation .	17	59	43·00
Greenwich Apparent Time of transit	19	43	16·29
Local Apparent Time of transit	24		
∴ Difference of times	4	16	43·71

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104. We will now take some examples in which the particular

cases, already illustrated, do not occur, the triangle PZS being oblique-angled, its sides and angles being interpreted exactly as before.

1. The apparent altitude of a star, whose declination was $+33° 14' 0''$, was observed to be $47° 22' 54''$, when its azimuth was $106° 15' 0''$; find the time at which it would pass the meridian, the mean time of the observation being $14^h 15^m 21^s$.

Here $z=42° 38' 0''$; $\delta=56° 46' 0''$; $Z=106° 15' 0''$,

whence, by means of the equation $\sin H=\sin Z.\dfrac{\sin z}{\sin \delta}$, we find

$$H=51° 1' 21''=3^h 24^m 5^s{\cdot}4,$$

which is the sidereal interval which will elapse before the star comes to the meridian; the equivalent of this, in mean solar time, is the interval which the mean time clock used in the observation will mark between the observation and the transit. Applying this correction, as explained at art. 90., we have

	h	m	s
Sidereal interval	3	24	5·4
Retardation for 3·4 hours . .			33·4
Mean time interval	3	23	32·0
Mean time of observation . .	14	15	21·0
Mean time of transit	17	38	53

2. The apparent altitude of a star, whose declination was $+15° 7' 43''$, was observed to be $62° 17' 36''$, when its azimuth was $75° 15' 10''$; what was the latitude of the place of observation?

By formula (2.), p. 54, we get

$$\cos\delta=\cos z.\cos\lambda+\sin z.\sin\lambda.\cos Z;$$

whence, putting $\tan z.\cos Z=\tan\varphi$, we get

$$\cos\delta=\cos z.\frac{\cos(\lambda-\varphi)}{\cos\varphi},$$

$$\text{or}\qquad \cos(\lambda-\varphi)=\cos\delta.\cos\varphi.\sec z,$$

from which, when $\lambda-\varphi$ has been found, we may find λ, since φ is already determined.

In the question, $z=27° 42' 55''$; $\delta=74° 52' 17''$; $Z=75° 15' 10''$; whence we find the latitude $=9° 22' 24''$.

3. At $4^h 23^m$, Woolwich mean time, on May 19, 1852, the sun was observed to cast no shadow in front or back of the Royal Military Academy; in what direction does the building face, its latitude

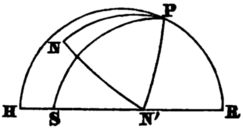
being 51° 28′ 28″ N., the sun's declination 19° 44′ 21″ N., and the equation of time $+3^m\ 49^s$?

Building faces N. 5° 20′ 36″·5 W.

4. At a certain time, when the Æ and declination of Sirius were $6^h\ 38^m\ 36^s$ and −16° 30′ 49″, and of Procyon $7^h\ 31^m\ 31^s$ and +5° 36′ 10″, it was observed that the true setting of Sirius took place $3^h\ 42^m\ 10^s$ (sidereal) before that of Procyon. What was the latitude of the place?

Let N be the position of Procyon when Sirius sets at S, and let N′ be the setting place of Procyon; then NPN′ is the difference of the times of setting, and NPS the difference of the Æs; so that we readily find N′PS; and having PN′ and PS, we can find the angle PSN′ or the angle PN′S. In the first case we can find PR the latitude, since PRS is a right angle, and we know PS the polar distance of Sirius; or, in the other case, we find the supplement PN′R, whence, having also PN′ in the right-angled triangle PN′R, we find PR the latitude. Using this last method, we find PN′R = 61° 23′ 6″, and thence PR = 60° 53′ 14″.

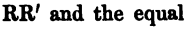
Fig. 26.



5. The altitude of Regulus was observed to be 16° 31′ 5″ at $9^h\ 15^m\ 0^s$, and at $11^h\ 43^m\ 8^s$ it was again observed and found to be 40° 11′ 27″; what was the latitude of the place of observation, the declination of Regulus being +12° 41′ 30″?

Let Z be the zenith, P the pole, R and R′ the two positions of Regulus; and we have PR = *PR′ = 77° 18′ 30″; ZR = 73° 32′ 10″; ZR′ = 49° 49′ 43″, and the angle RPR′ = 37° 0′ 2″; whence we find PRR′ = 85° 47′ 28″; RR′ = 36° 6′ 2″; ZRR′ = 42° 34′ 36″; PRZ = 43° 12′ 52″;

Fig. 27.



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* The equality of these sides causes a useful simplification in the formulæ (4.) and (6.) page 55, for the determination of the side RR′ and the equal angles at R and R′; for calling the sides r and r, we get

$$\tan\tfrac{1}{2}(\mathrm{R}+\mathrm{R}') = \frac{\cos\frac{1}{2}(r-r)}{\cos\frac{1}{2}(r+r)}\,.\cot\tfrac{1}{2}\,\mathrm{P},$$

or $$\tan\mathrm{R} = \sec.r.\cot\tfrac{1}{2}p;$$

and $$\tan\tfrac{1}{2}(r+r) = \frac{\cos\frac{1}{2}(\mathrm{R}-\mathrm{R})}{\cos\frac{1}{2}(\mathrm{R}+\mathrm{R})}\,.\tan\tfrac{1}{2}\,\mathrm{RR}',$$

whence $$\tan\tfrac{1}{2}\,\mathrm{RR}' = \tan r.\cos\mathrm{R}.$$

and finally, $PZ = 41° 55' 4''$;
whence we get the latitude $= 48° 4' 56''$.

6. At $17^h\ 51^m\ 6^s$, Greenwich mean time, the altitude of the sun's lower limb was observed, after it had passed the meridian, to be $62° 58' 54''$, and the azimuth of the east limb $112° 32' 0''$; the declination of the sun's centre was $+15° 27' 40''$, its semidiameter $15' 55''$, and the equation of time $+3^m\ 7^s$; find the latitude and longitude of the place of observation.

Applying the corrections for refraction and semidiameter, we get

$$z = 26° 45' 20'', \text{ and } Z = 111° 56' 38'';$$

whence, with the remaining data, we find

Latitude = 27° 43′ 50″ N.
Longitude = 117° 7′ 10″·5 E.

7. In south latitude 30° 17′ 10″, the altitude of β Leonis, before coming to the meridian, was observed to be 17° 45′ 9″, and its azimuthal distance from a distant terrestrial object to the westward of it was 60° 26′ 23″; what was the angular distance of that object from the meridian, the declination of β Leonis being +15° 24′ 14″?

Azimuth of object S. 1° 3′ 26″ W.

8. At a place in Canada, the altitude and azimuth of α Lyræ, before passing the meridian, were observed, at $5^h\ 14^m\ 10^s{\cdot}6$ Greenwich mean time, to be, altitude 15° 30′ 59″, azimuth 47° 42′ 40″. Find the latitude and longitude of the place of observation; and thence determine its distance from Quebec (in latitude 46° 49′ N., longitude 71° 16′ W.), and also the bearing of Quebec. The Æ and declination of α Lyræ being 18° 31′ 55″ and 38° 39′ 14″; the Æ of the sun $11^h\ 6^m\ 18^s{\cdot}5$, the equation of time $+2^m\ 24^s$; and the radius of the earth 3956 miles.

	°	′	″	
Latitude	49	21	52	N.
Longitude	81	49	3	W.
Bearing of Quebec N.	105	53	25	E.

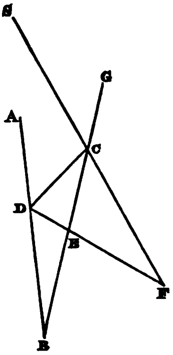
Distance 516·88 miles.

ON THE DETERMINATION OF TIME BY LUNAR DISTANCES.

Before proceeding to investigate the method of determining time by the measurement of the direct distance of a fixed star from the Moon, it will be well to describe the mode in which the properties of reflected light are made use of in measuring angular distances, which, under certain circumstances, at sea for instance, could be measured in no other way.

105. The fundamental law of reflexion, viz. that "the angle of incidence is always equal to the angle of reflexion" (art. 11.), leads at once to the property which forms the principle of construction of the instrument we are about to describe. This property may be thus enunciated:—"*When a ray of light is successively reflected from two plane surfaces, the angle between its course before the first reflexion, and its course after the second reflexion, is the double of the angle of inclination to one another of the two reflecting surfaces.*"

Fig. 28.



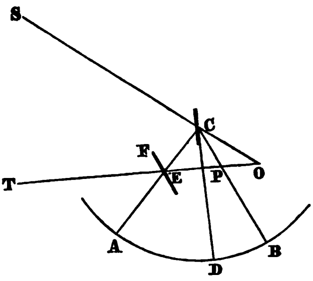
Let AB and BG be two reflecting surfaces; and SCDE the course of a ray of light, reflected successively from them; produce SC, DE to meet at F: then shall $SFD = 2ABC$

for $ECF = GCS = DCE$ (by incidence and reflexion), and we have

$$\begin{aligned} SFD &= DEC - ECF \\ &= EBD + BDE - ECD \\ &= EBD + ADC - BCD \\ &= EBD + EBD = 2ABC. \end{aligned}$$

106. To show how this principle is applied to the measurement of angles, let CB, CA be two bars fixed at an angle ACB, and connected at their other extremities by a graduated arc AB, whose centre is C; CD a third bar moveable in the plane ACB, about C as a centre, and bearing at C a small mirror, whose plane is parallel to CD, and perpendicular to the plane ACBD; and at E let another such mirror be placed, having its plane also perpendicular to the plane ACB, but parallel to CB, and having one half of its surface unsilvered, so as not to obstruct a ray proceeding in the direction TE to the eye of the observer at O; then if SO be the direction of a ray coming from another object, it may always be reflected from the mirror at C in the direction CE, by simply turning the bar CD about its axis of motion at C, and therefore always be reflected

Fig. 29.



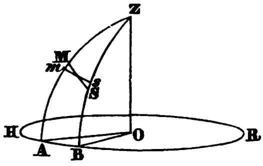
from the lower part of the mirror at E, in the direction EO; the consequence of this is, that while the object S is seen by reflexion in the direction OE, the object T is seen through the transparent part in the same direction, that is, to the eye at O, the two objects appear to coincide. When this is the case, we have the angle SOT equal to twice the inclination of EF to CD, or of BC to CD (since EF is parallel to BC), that is, to twice the arc BD; if, therefore, BA be graduated from B towards A, so as to measure twice as many nominal degrees as it really contains, the reading of the arc BD will give us the measure of the required angle SOT.

The combined rays, passing from E towards O, are usually received by the eye through a small telescope at P; if then the object S be not so distant as to render the angle SPT immaterially different from the angle SOT, that is, render it immaterial whether the object be viewed from P or from O, some allowance must be made for the faulty position of the eye at P; but in practice this correction is, except for very close objects, so small, that, after this caution, no error is likely to arise in the use of the instrument; when applied to astronomical purposes this source of error altogether vanishes, if only care be taken to view any terrestrial object T, with which it may be requisite to compare a star seen in the direction SC, through the transparent part of the glass EF, that is, by direct vision.

107. The great advantage of this instrument, which is called "*The Sextant*," consists in its not requiring a fixed stand; it may be, and generally is, held in the hand of the observer, and it is turned about until the plane of its "*face*" passes through both the objects under observation; the moveable radius CD is then shifted until both objects are seen in the direction PE, when it only remains to "*read off*" the number of degrees, minutes and seconds, graduated on the arc BD, to complete the observation. All this may be done on the deck of a ship at sea, notwithstanding a considerable amount of rolling motion, the least symptom of which would render any other astronomical instrument useless; and the sea horizon is almost always available as a circle of reference for the determination of altitudes.

It has been hinted that the observed angular distance of the moon from certain fixed stars may be made use of for the determination of the Greenwich mean time, when this distance is not estimated by the difference of the ARs of the two bodies, measured by means of the observed difference of their times of transit over the meridian of the place whose longitude it is wished to determine, but by the arc of a great circle passing through them. This "*Lunar distance*," as it is called, can be measured by means of the sextant; but when thus measured, it is affected, to a greater or less amount, with the errors arising from refraction and parallax, and in this state it is improper to be compared with the distances tabulated in the Almanac, which are the true distances which would be observed from the centre of the earth; if, however, the altitudes of the two bodies be observed at the same instant as their distance, we can obviate this source of error, by applying the requisite corrections to these observed altitudes, and can thence obtain the true distance. Thus, let M and S be the true places of the moon's centre and the star, as seen from the centre of the earth; *m*, the apparent position of the moon's centre (being raised by the effect of refraction, and lowered by parallax); *s*, the apparent position of the star; then MS is the true distance, and *ms* the apparent or measured distance; *m*A, *s*B the apparent altitudes; MA, SB the true altitudes, found from the former, by applying to them the requisite corrections. Then in the triangle *m*Z*s* we know *m*Z, *s*Z and *ms*, and can therefore find the angle *m*ZS, which is the same as MZS, since the effects of both parallax and refraction take place in the vertical planes AZO, BZO. We have then in the triangle MZS, MZ, ZS, and the angle MZS, to find MS the true distance.

Fig. 30.



108. If, instead of measuring the apparent distance *ms*, we observe the difference of azimuth AB, which measures the angle Z, we obviate the necessity of calculating that angle from the sides of the triangle *m*Z*s*; the former method, however, is that which must be adopted at sea, as the difference of azimuth cannot then be measured with sufficient accuracy. Let z and z' be the *apparent*, Z and Z′ the *true* zenith distances; *d* the *apparent* and D the *true* distance between the star and the moon's centre; A the difference of their azimuth, that is, the angle MZS; then if 2σ be the sum of the three sides of the triangle *m*Z*s*, we have

$$\tan\frac{1}{2}A=\sqrt{\frac{\sin(\sigma-z)\sin(\sigma-z')}{\sin\sigma.\sin(\sigma-d)}},$$

and we shall have, for the determination of D, either

$$\cos D=\cos Z.\cos Z'+\sin Z.\sin Z'.\cos A, \quad . \quad . \quad . \quad (1.)$$

or (art. 99.) $\cos D=\cos Z.\dfrac{\cos(Z'-\varphi)}{\cos\varphi}$, where $\tan\varphi=\tan Z\cos A$ (2.)

We may, however, slightly facilitate the calculation of D in those cases in which the angle A is not obtained by direct observation, in the following manner:—

Since $\cos A=\dfrac{\cos D-\cos Z.\cos Z'}{\sin Z.\sin Z'},$

and $\cos A=\dfrac{\cos d-\cos z.\cos z'}{\sin z.\sin z'},$

by subtracting these severally from unity and equating the results, we get

$$\frac{\cos(Z-Z')-\cos D}{\sin Z.\sin Z'}=\frac{\cos(z-z')-\cos d}{\sin z.\sin z'}$$

$$=\frac{2\sin\frac{1}{2}(d+z-z')\sin\frac{1}{2}(d+z'-z)}{\sin z.\sin z'}$$

$$=\frac{2\sin(\sigma-z').\sin(\sigma-z)}{\sin z.\sin z'},$$

whence $\cos D=\cos(Z-Z')-\dfrac{2\sin(\sigma-z').\sin(\sigma-z).\sin Z.\sin Z'}{\sin z.\sin z'}$

$$=\cos(Z-Z')-\cos X,$$

putting cos X for the last term of the preceding; or, as before, if

$$S=\frac{1}{2}(X+Z+Z'),$$

$$\cos D=2\sin(S-Z).\sin(S-Z') \quad . \quad . \quad . \quad . \quad (3.)$$

109. Having found the true distance, D, it only remains to interpolate it between those given in the almanac, which are at such near intervals as to afford a tolerably close approximation to the Greenwich time at the moment of observation, by simple proportion, which assumes that the motion of the moon, during the interval from one tabulated distance to the next, is uniform; this, although not accurately true, is sufficiently so for our present purpose. The altitude of the star will afford the means of obtaining

the local time, which, compared with that of Greenwich, will give the longitude.

Examples.

1. The observed altitude of the moon's lower limb was 34° 17′ 22″, when that of α Arietis was 42° 13′ 47″ (to the east of the meridian), and their measured distance 21° 3′ 18″; the moon's bright limb being towards the star. The moon's horizontal parallax was 54′ 15″ and her semidiameter 14′ 49″; find the longitude of the place of observation, supposing the distances nearest to the above, tabulated in the almanac, to be, at 3ʰ Greenwich mean time, 20° 22′ 21″; at 6ʰ Greenwich mean time, 21° 27′ 54″; and that the local mean time of the observation was 15ʰ 27ᵐ 31ˢ.

Hor. Par. of moon ...... 54′ 15″=3255″ .... 3·5125510 } (art. 44.)
App. Z. D. moon's lower limb 55° 42′ 38″ ...... sin 9·9170863 }
Semidiameter 14 49
∴ z=55 27 49
Parallax in altitude ....= 44 50 .... 3·4296373
54 42 59
Refraction....................... 1 25
True Z. D. moon's centre ... 54 44 24=Z.

App. Z. D. of * .... 47° 46′ 13″=z'. Dist. * from moon's bright limb 21° 3′ 18″
Refraction .... 1 6 Semidiameter 14 49
True Z. D. of * .... 47 47 19=Z′. App. dist. * from moon's centre 21 18 7=d.

z = 55° 27′ 49″
z'= 47 46 13
d= 21 18 7
2)124 32 9
σ= 62 16 4·5 ac. sin. 0·0529914
$\sigma-d$= 40 57 57·5 ac. sin. 0·1833540
$\sigma-z$= 6 48 15·5 ... sin. 9·0736398
$\sigma-z'$= 14 29 51·5 ... sin. 9·3985304
2)18·7085156
12 44 20·7 ... tan. 9·3542578
2
∴ A = 25 28 41·4 ... cos. 9·9555671
Z = 54 44 24 ... tan. 0·1505856 ... cos. 9·7613923
φ = 51 56 1 ∴ tan. 0·1061527 ac. cos. 0·2100147
Z′= 47 47 19
φ−Z′= 4 8 42 .... cos. 9·9988625
∴ D=20° 57′ 34″ ... cos. 9·9702695

Distance at 3^{h} Greenwich mean time	20 22 21		
Distance at 6 Greenwich mean time	21 27 54		
Change for required interval	0 35 13	= 2113 sec ...	3·3248995
Change for 3 hours' interval	1 5 33	= 3933 ac.	6·4052761
		3 hours ...	0·4771213
	$1^{\text{h}}\ 36^{\text{m}}\ 42^{\text{s}}·3$	... 1·611747 ...	0·2072969
	3		
Greenwich mean time of observation	4 36 42·3		
Local mean time	15 27 31·0		
Longitude East	10 50 48·7		

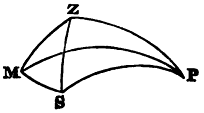
Let us now calculate D by formula (3.) :—

d	=	21 18 7		
z	=	55 27 49	... ac. sin.	0·0841960
z'	=	47 46 13	... ac. sin.	0·1305007
		2)124 32 9		
σ	=	62 16 4·5	log 2 =	0·3010300
$\sigma - z$	=	6 48 15·5	... sin.	9·0736398
$\sigma - z'$	=	14 29 51·5	... sin.	9·3985304
Z	=	54 44 24	... sin.	9·9119779
Z′	=	47 47 19	... sin.	9·8696254
X	=	86 37 41·3	... cos.	8·7695002
		2)189 9 24·3		
S	=	94 34 42·1	log 2 =	0·3010300
S − Z	=	39 50 18·1	... sin.	9·8066032
S − Z′	=	46 47 23·1	... sin.	9·8626359
∴ D	=	20 57 34·6	... cos.	9·9702691

The work by this method is more compact and less troublesome than by the former.

110. Having thus obtained the true distance, and thence the Greenwich mean time at which the observation was made, we may, if desirable, find the latitude as well as the longitude of the place of observation, even without previously knowing the local mean time; for the Greenwich mean time will enable us to find in the almanac, the Æ and declination of the moon (which is there given for every hour throughout the year), and also those of the star or the sun, if the latter be the other object observed. By means of these values, if Z (fig. 31) be, as before, the zenith, and P be the pole, we have, in the triangle PMS, PM, the polar distance of the moon, PS that of the star, MS the true distance, and the angle MPS, the difference

Fig. 31.



of the Æs of the two bodies, which data are more than sufficient to enable us to determine the angle MSP or the angle SMP: and in the triangle MZS, having also the three sides and the angle MZS (=A, in the previous calculation), we can find ZSM or ZMS; finally, in the triangle PSZ, we have PS, SZ and the angle PSZ (=PSM−ZSM) from which to determine PZ the co-latitude, and ZPS, the hour-angle; or in the triangle ZMP, we have ZM, MP and the angle ZMP (=ZMS−PMS) from which to find the colatitude PZ and the hour-angle ZPM. By means of the hour-angle ZPS combined with the Æ of the moon and that of the sun, we readily find the local mean time, and thence, as before, the longitude.

111. But if, as in the question, the local mean time be observed, we can with the proper values of the Æ of the star or moon and that of the sun, obtain the value of the hour-angle ZPS or ZPM, whence, having also, in the triangle ZPS, the two sides ZS and SP, we can find PZ; or in the triangle PZM, having ZM and MP, we can also find PZ.

Examples.

2. The apparent altitude of the sun's lower limb was 34° 5′ 44″, and of the moon's lower limb 56° 56′ 29″, when the apparent distance between their nearest limbs was 35° 16′ 40″. Find the true distance between their centres; the moon's horizontal parallax being 54′ 15″, and that of the sun 0′ 8″.

True distance 35° 59′ 14″.

3. The apparent altitude of a star was 20° 13′ 26″, and that of the moon's upper limb 31° 32′ 16″, when the apparent distance of the star from the moon's bright limb (which was on the side next the star) was 72° 27′ 22″. The semidiameter of the moon was 14′ 54″, and her horizontal parallax 54′ 26″. Find the true distance of the star from the moon's centre.

True distance 72° 33′ 4″.

4. The apparent distance of the nearest limbs of the sun and moon was observed to be 80° 29′ 40″, when the altitude of the sun's lower limb was 72° 9′ 50″, and that of the moon's upper limb 18° 46′ 38″: find the true distance of their centres, the semidia-

meter of the sun being 16′ 10″, that of the moon 16′ 38″, and her horizontal parallax 60′ 58″; that of the sun being as in question 1.

True distance = 80° 9′ 34″.

5. The measured distance of a star from the moon's bright limb, which was away from the star, was 18° 37′ 58″, when the zenith distance of the star was 46° 33′ 10″, and that of the moon's lower limb 58° 47′ 23″: what was the true distance, the semidiameter of the moon being 16′ 23″, and her horizontal parallax 60′ 21″?

True distance = 19° 49′ 10″.

6. The distance between a certain star and the moon's centre was observed to be 27° 35′ 0″, when the altitude of the star was 29° 47′ 0″, and that of the moon's centre 57° 22′ 0″; what was the true distance, the horizontal parallax of the moon being 60′ 3″?

True distance = 28° 8′ 24″.

7. The apparent altitude of the moon's centre was observed to be 16° 26′ 0″, when that of Venus was 29° 41′ 0″, and their measured distance 98° 15′ 31″; the moon's horizontal parallax was 60′ 35″ and that of Venus 20″; find the true distance.

True distance = 97° 45′ 4″.

8. The measured distance of Jupiter from the moon's centre was 120° 18′ 46″, when the altitude of Jupiter was observed to be 8° 26′ 0″, and that of the moon's centre 19° 24′ 0″; the moon's horizontal parallax being 57′ 14″ and that of Jupiter 1″·5; find the true distance.

True distance = 120° 1′ 45″.

CHAPTER V.

ON INSTRUMENTAL ADJUSTMENTS.

The Vernier.—Mode of determining the value of vernier divisions.—The Level —Mode of adjustment.—Wires of the telescope.—Mode of determining the correction for the zenith-point.—Line of collimation.—Error of collimation. —Adjustment of the Transit Instrument to the plane of the meridian.— Adjustments of sextant.—The artificial horizon.—Dip of the sea horizon.— Concluding remarks.

THE foregoing pages will be practically useless, unless something more than the mere theory of instrumental measurement of astronomical arcs be laid before the student; it is proposed, therefore, in the present chapter, to explain the practical means provided in every such instrument for obtaining measures which are as nearly correct as the particular instrument will allow.

112. The angular measures with which we have exclusively to deal, are necessarily dependent upon the accuracy with which the circular arcs or "*limbs*" of the instruments employed are "*divided,*" marked off, that is, in degrees, &c., and this accuracy will, other circumstances remaining the same, be greater the larger the radius of the circle to be divided, because the length of each degree is proportional to the length of the radius, and the longer the degree, the greater number of equal parts into which it can be accurately divided.

113. When these subdivisions have, in any instrument, been made as numerous as is considered expedient, the beautiful contrivance, called, after its inventor,

"THE VERNIER,"

comes to our further assistance, by adapting to our want the principle, that if two equal magnitudes be divided, the one into $n-1$ equal parts, and the other into n equal parts, each part of the first must exceed each part of the second by one nth of each part into which the first has been divided.

114. To make this clear, let a and b be the values of the respective parts, then

$$(n-1)a=nb;$$

$$\therefore\ b=\frac{n-1}{n}a$$

and $$a-b=a-\frac{n-1}{n}a=\frac{a}{n},$$

that is, the excess of a above b is equal to one nth part of a.

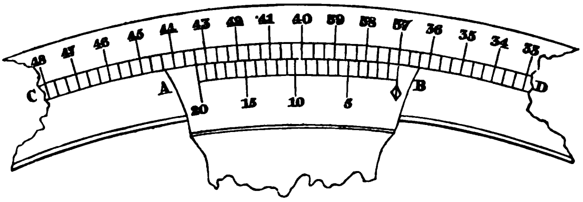
In order to show how this principle is applied to our use in the measurement of arcs, let us suppose that each degree on the limb of our instrument is divided into *three* equal parts, so that each part is equivalent to 20′: then, if an arc consisting of 19 of these parts be divided on another limb, having the same length of radius into 20 equal parts, it follows, from the above formula, making $n=20$, that each division on the first limb exceeds each division on the other by $\frac{1}{20}$ of the former parts; or, since each of the former parts is equivalent to 20′, the above excess is represented in minutes by

$$\frac{1}{20}\text{ of }20'=1'.$$

So that the excess of each of the former parts over each of the latter is 1′ of arc.

Let now the short limb or *vernier* AB slide on the limb CD of an instrument by a motion on the common centre of the two, and let them be divided as above; then if, as in the figure, the first division of the vernier be opposite the first division of the limb CD, the space between the second of the vernier and the second of the limb is an arc of 1′; the space between the third of the vernier and the third of the limb is an arc of 2′, and so on; the space between the twenty-first or last of the vernier, and the twenty-first of the limb being, as we know, an arc of 20′. If, then, we found in an

Fig. 32.



instrument so divided, that instead of the first division of the vernier coinciding with any division of the limb, it happened to be the fourteenth, for instance, we should at once know that the space from the first division of the vernier to the immediately preceding division of the limb would be equivalent to 14′ of arc.

Again if 59 parts on the limb so divided were equivalent to 60 on the vernier, we should have, as the excess of the parts of the limb over those of the vernier,

$$a-b=\frac{1}{60}\times 20'=20'';$$

so that, by means of a vernier so constructed, we should be able, on a limb divided to 20′, to "*read*" an arc to 20″.

In order then to determine the reading of any particular instrument, we have only, as in the figure, to place the vernier so that the zero-point, marked ⫰, coincides with any convenient division of the limb, and count the number of divisions of the vernier, say *n*, up to that one which next coincides with one on the limb, which will generally be the last; then each division of the vernier will correspond in reading to one *n*th of each part into which the degrees are divided on the limb.

115. There is a still more delicate mode of subdividing the parts on the limb, made use of in the larger instruments, employed in fixed observatories, viz. the "*micrometer microscope*;" but, as it is beyond the proposed scope of the present work to describe the various delicate adaptations in such instruments, this, as well as others not ordinarily provided in instruments of moderate size, will not be further alluded to.

116. The ALTITUDE AND AZIMUTH INSTRUMENT with the TRANSIT INSTRUMENT, which is, in fact, but a restricted modification of the former, are the only instruments besides the SEXTANT which we need here describe the mode of using. It is evident from the purposes to which we have shown the first of these to be applied (art. 55.), that the primary considerations to be attended to, are the verticality of the axis of its horizontal motion and the horizontality of the axis of its vertical motion. These are both secured by the use of the

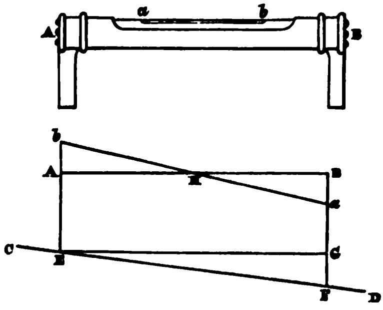
"SPIRIT LEVEL,"

which is constructed upon the principle that, at any point of the earth, the surface of a fluid will, if left to itself, be parallel to the general figure of the surface of the earth at that point, and therefore, that a small portion of such a surface may be considered as a plane surface tangential to the surface of the earth, or perpendicular to the direction of the plumb-line at that point.

117. If, then, a small quantity of fluid be enclosed within a cylindrical glass tube, which it nearly fills, a bubble of air being alone left, when the tube is parallel to the tangent plane, the ends of the bubble will both appear on the same side of the cylinder, and if the cylinder be slightly curved upwards in the middle, the bubble will seek the highest point where alone the surface of the fluid can be parallel to the tangent to the curve which the cylinder forms at that point, or, which is the same thing, parallel to the tangent plane of the earth's surface. In practice, this highest point is always considered to be the middle, and, when the level is properly adjusted and the bubble in the middle, the extreme points upon which the ends of the level are supported, are what is termed "*level.*"

118. This little instrument is, like all others, liable to get out of adjustment, through accidental straining of the parts, or otherwise; that is, its indications are no longer to be depended on, until they have been verified by some test. Let AB be the line joining the two ends of the level, which is supported upon the two legs AE, BF, then, if the bubble be in the middle, AB is horizontal; but it does not follow that CD, the surface upon which the legs rest, is so likewise. In order to try this, however, we have only to "*reverse*" the level, interchange, that is, the positions of the two legs; if this be done in our figure by making E*b* equal to FB and F*a*=EA, then, if the legs AE, BF be of unequal length, we see at once that the new position *ab* of the level must be inclined at an angle *b*HA to the true horizontal AB, which angle depends entirely upon the inequality of the two legs, because, had they been equal, the bubble could only have appeared in the middle of AB when CD was parallel to it, that is, also horizontal. The question then remains, how is this inequality of the legs to be corrected? If *b*E be shortened

Fig. 33.

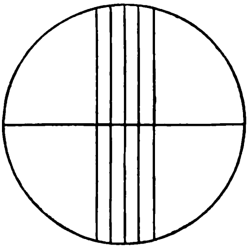


until the bubble is again in the middle, it is clear that too much will have been done, since we shall be just in the same position from which we started; but if *b* be lowered to A by shortening the leg *b*F, and *a* be raised to B by raising the supporting surface CD until F coincides with G, where the horizontal line through E meets BF, then the legs will be equal, and the supporting surface horizontal. This process is always provided for by supporting the surface CD upon screws, by turning which, one end or the other, may be raised or depressed; the level also is similarly provided, unless so constructed that nothing short of violent usage can interfere with its correctness.

119. In the altitude and azimuth instrument two levels are attached at right angles to one another upon the revolving horizontal plate HR, which carries the index in our figures (pages vi. vii.); so that the lower plate LM supported upon the tripod screws F, F, F being raised or depressed on the one side or the other, as the case may require, by means of these screws it may, by the help of the levels, be placed in a horizontal position, thus ensuring the verticality of the axis of horizontal motion. If it should be found that the indications of the levels do not remain the same when the instrument is turned round horizontally upon its axis 180°, when the levels will be reversed, the error, upon the principle we have laid down, must be corrected by depressing the higher end of the level, by means of its screw adjustment, and at the same time raising the opposite part of the horizontal plate by one of the screw feet. In a very similar manner the horizontality of the axis of vertical motion is effected by means of a detached "*striding*" level, which rests its legs upon the ends of the axis, and thus *strides* over the intermediate part of the instrument.

120. The telescope attached to this instrument has usually one horizontal, and three, or more, vertical wires so placed in the interior, as to be clearly visible in the sky, drawn as it were across the small portion of the heavens which forms the "*field*" of the telescope (fig. 31). These wires are illuminated at night by a lamp which throws its rays in at one end of the horizontal axis, to the centre of the telescope, where a polished flat elliptical ring,

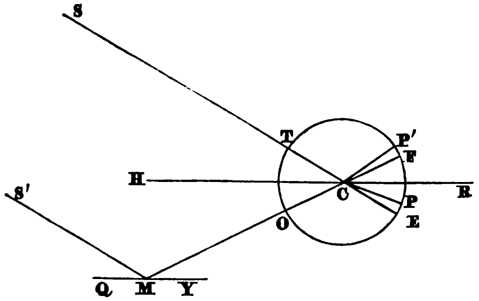
Fig. 34.



placed at an angle of 45° to the axis of the telescope, reflects them towards the eye-glass, where the wires are placed. These wires are the means by which the positions of objects are observed; the horizontal one enables us to observe altitudes, while the centre one of those placed vertically is used in the measurement of azimuths, and, combined with the others parallel to it, in the observation of transits. Now the accuracy of the first of these measures (altitudes) depends entirely upon two facts, the horizontality of the axis of vertical motion, which we have already shown how to secure, and the identity of the direction of this horizontal wire with that of the zero or starting-point of the graduations of the vertical circle which is attached to, and moves with the telescope, because, to whatever amount these two directions differ, the altitudes taken with the instrument will be erroneous.

121. The mode of determining the amount of this deviation, which presents perhaps least difficulty, is to observe the altitude of any star when on the meridian, and immediately, by turning the telescope down, observe the reflexion of the same star seen in a small trough of mercury; these observed arcs, the one of elevation, and the other of depression, should be equal; for let STE and SM be the parallel directions of two rays from the same star; HR a horizontal line through the centre of motion C; ET the direction of the telescope at the first observation; FO its direction when the ray reflected from the surface QY of the mercury at M is observed; then, since the angle SMQ is equal to YMF, and the former is equal to SCH, while the latter is equal to HCM, it follows that FCR is equal to RCE. If, then, it be found that these two angles differ as measured on the graduated circle, such difference must

Fig. 35.



arise from the zero-point of the vertical circle not being at the point F or E; in fact, if we for the moment assume that it is at the point P, transferred to P′ when E is transferred to F at the second observation, we see that the measured angle PCR will be as much in defect of the true value ECR as the other measure P′CR is in excess of the value FCR, which is equal to the former. So that the smaller measure, the true value, and the greater measure form an arithmetic progression, and therefore the true value will be obtained by taking half the sum of the two measured arcs and the "*index error*" for zenith distances, or, as it is also called, the correction for the zenith point, that is, the angle PCE will be found by taking half their difference.

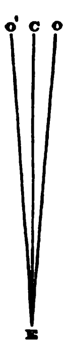
122. There are usually means provided for getting rid of this error by shifting the vernier upon its supports, but unless it should be of inconveniently large amount it is better not to meddle with them, but rather to correct each observation by the amount of index error, at the same time that the corrections for refraction and parallax are applied. The index error may also be obtained by "*reversing the instrument,*" that is, by first observing the altitude with the "*face,*" or graduated side of the vertical circle towards the east, and then with the face towards the west; if there be any difference in these readings, it must manifestly be due to an incongruity between the positions of the zero-point of the graduations and of the horizontal wire; and again, half the difference of these readings will give the index error.

123. We now come to the consideration of the position of the middle vertical wire, which ought to be fixed at right angles to the axis of the telescope, in the plane which is perpendicular to the axis of vertical motion. The straight line which joins the centre of the eye-glass and the middle wire is called the "LINE OF COLLIMATION" of the telescope, and the axis of the telescope may be considered to be the straight line drawn through the centre of the eye-glass perpendicular to the axis of vertical motion; and it is essential that these lines should coincide, because if they did not, the middle wire would, as seen through the telescope, describe a small circle in the heavens, as the telescope turned upon its axis of vertical motion; and consequently, supposing it to be seen in the direction of any particular vertical circle when the telescope is in a certain position, it would

depart more and more from that circle as the telescope is made to move on its axis of vertical motion.

124. Any "*error of the line of collimation*" may be detected on the same principle as that involved in the determination of the index error, and by means similar to those last pointed out for that purpose; for if a distant terrestrial object be bisected by the vertical wire and the reading of the horizontal limb be marked, and if the instrument be then carefully turned half round in azimuth upon its vertical axis, and the object be again bisected by the middle wire, then, if there be any error in the line of collimation, the reading of the horizontal limb will differ from the previous reading, always supposing that the readings on the opposite verniers of the horizontal circle correspond; for let EO be the axis of the telescope, and EC the line of collimation at the first observation, then by turning the instrument half round in azimuth, and bringing the middle wire again to bear upon the same object, the line of collimation is transferred to the other side of the axis, or, which amounts to the same, the axis is transferred to the other side of the line of collimation, and will therefore take the new position EO′, making the angle O′EC equal to OEC. The difference of the two readings, then, will be the angle O′EO, the half of which, viz. O′EC, will be the arc through which it is necessary to move the wires in order to make the line of collimation coincide with the axis of the telescope.

Fig. 36.



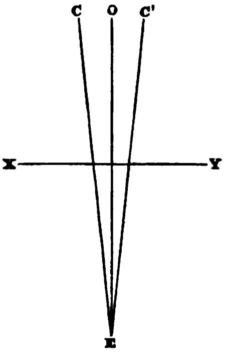
This error can only be corrected by an actual horizontal shifting of the frame which holds the wires, so as to bring the middle wire into a correct position without disturbing the distances of the other wires from it; this can be effected by means of side screws provided for this purpose, which act upon the frame holding the wires; viz. by loosening the screw on one side and tightening that on the other, which should be done very carefully, as an accident to this part of the instrument renders it quite useless. The observations and shifting of the wires must be repeated until the error disappears.

125. We now proceed to point out the use of the other vertical wires, placed by the maker parallel to the middle one. These are in number either one, two, or three, on each side of the middle wire, according to the size of the instrument, and their common vertical-

ity, and therefore parallelism, can, like that of the middle wire, be determined by observing whether, as the telescope moves in a vertical circle, the same object continues to be bisected as each point of the particular wire passes it. We will assume that our instrument has five wires; these should be equidistant from one another, so that, when the instrument is in the meridian, that is, when the circle traced out by the middle wire coincides with the meridian, the interval of time which elapses between the instants of transit of the same body over the five wires shall be equal. This equality of the intervals being secured by the maker, it will follow that the times of the five transits of the same body over the wires will form an arithmetic progression, and therefore the mean of the whole should be the same as the time of transit over the middle wire; and the advantage of the arrangement is not simply confined to its thus forming a check to the accuracy of the middle observation, for it really increases that accuracy, because, as the body will rarely appear on any particular wire at any beat of the clock, it is requisite to estimate the fractional parts of a second between the last beat of the clock and the instant of the star's transit; this, by practice, can be done very accurately, but, as it is at best but guess, it is of importance to diminish the chances of error by the fivefold repetition of the observation which is effected by means of the five wires.

126. When placed in the plane of the meridian, the altitude and azimuth instrument answers every purpose of the transit instrument, the adjustments of which, viz. for level of axis and for line of collimation, are made on the same principle as in the former instrument; the latter adjustment, however, as the instrument is unencumbered by the large vertical circle, is, in small instruments, tested by actual reversion of the telescope, by taking it out of its supports and carefully turning the axis end for end, when, if it be found that the same object is not bisected, one half of the apparent deviation must, as before, be corrected by an actual shifting of the wires. For, let EO be the axis of the telescope perpendicular to the axis of motion XY, and let EC be the line of collimation; then, if the axis be reversed, we shall have

Fig. 37.

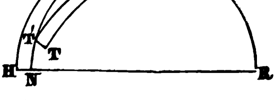


the line of collimation in the direction EC′, since the direction EO will be unaltered; so that the middle wire, instead of covering an object in the original direction EC, will now cover one in the direction EC′, the angle between which and the former direction, viz. the angle CEC′, is the double of OEC′, the deviation of the line of collimation from the axis of the telescope.

127. In the text, and in some of the examples we have pointed out some of the methods by which the altitude and azimuth instrument may be placed in the meridian by correcting, on the azimuth circle, the deviation which calculation shows to exist in the position of the vertical circle in which the axis of the telescope moves. These methods are not applicable to the transit instrument, because they, more or less, depend upon the azimuthal motion of the instrument, a motion which the transit instrument possesses only to the very limited degree sufficient to correct the small amount of deviation from the meridian which is sure to exist at first setting up the instrument after its other adjustments are perfected. One of the most useful methods of determining the deviation of this instrument from the plane of the meridian, is what is called the "*method of high and low stars*;" it consists in observing the transits of two stars differing but little from one another in right-ascension, but very considerably in declination, at least 40°. The difference of the times of transit should of course give the difference of the right-ascensions of the stars, if the motion of the middle wire were in the plane of the meridian, and if therefore these values are found not to be identical, there can be no doubt that the instrument deviates to a greater or less extent from its correct position.

128. The amount of this deviation may be thus investigated. Let RPZH be the meridian, P the pole, Z the zenith, ZSN the altitude circle traced out by the middle wire of the telescope, then the angle HZN is the "*deviation*": let S be the position of the upper star at its passage across the middle wire, and at the same instant let the lower star be at T, its observed point of transit being T′; then SPT is the difference of the ARs of the two stars, and T′PT the difference of their times of transit: if, therefore, the latter difference be subtracted from the former, the result will be the value of the angle SPT′. And since the deviation may always

Fig. 38.



be considered as of very small amount, ST′ the difference of the zenith distances will not sensibly differ from the difference of the polar distances, which it would be identical with if ZN were the true meridian; let, then, $PS=\delta$; $PT=\delta'$; $SPT'=P$; $ZP=\lambda$; the deviation $HZN=Z$, and we have in the triangle SPT′, $ST'=\delta'-\delta$, and consequently by formula (1), p. 54,

$$\sin PSZ = \sin PST' = \sin P \,.\, \frac{\sin \delta'}{\sin (\delta'-\delta)},$$

and, in triangle ZSP,

$$\sin Z = \sin PSZ \,.\, \frac{\sin \delta}{\sin \lambda} = \sin P \,.\, \frac{\sin \delta' \,.\, \sin \delta}{\sin (\delta'-\delta) \,.\, \sin \lambda};$$

or, if P and Z be estimated in seconds of arc, since they are both small,

$$Z = P \,.\, \frac{\sin \delta' \,.\, \sin \delta}{\sin (\delta'-\delta) \,.\, \sin \lambda}.$$

This expression for the deviation Z implies a knowledge of the colatitude of the place, λ, which however can always be very readily obtained to a sufficient degree of accuracy for this purpose, by an altitude of a known star observed on the approximate meridian ZN, by means of the small vertical circle attached to one end of the horizontal axis of the telescope for the purpose of directing it to the exact part of the meridian where the known star to be observed must culminate. It is easily seen that the magnitude of the angle P depends upon ST′ the approximate difference of the polar distances; if, therefore, this difference be not considerable, the angle P becomes so small that it is impossible to make the observation with sufficient accuracy to prevent its error having an overwhelming effect upon the deviation Z, increased as that effect is by the diminution of the divisor, *sin* $(\delta'-\delta)$.

129. It must be observed that in the above formula P is obtained by subtracting the difference of the times of transit from the difference of the right-ascensions, and that the polar distance δ belongs to the star which comes first to the plane of transit; and that, the deviation in our figure being to the eastward, we have derived a positive value for the correction Z: in the practical application therefore of the formula, we shall, if the above points be attended to, be safely guided by the algebraic sign of Z in our application of its value to the correction of the position of the instrument; viewing, that is, the deviation as being to the eastward when Z is positive, and to the westward when Z is negative.

As an example, let us suppose the following observations to have been made on 21 January 1853, in latitude 51° 28′ 28″.

Star.	First Wire.	Second Wire.	Third Wire.	Fourth Wire.	Fifth Wire.	Mean.
	m s	m s	h m s	m s	m s	h m s
Capella ...	5 26·7	5 44·4	5 6 2·3	6 20·1	6 38·1	5 6 2·32
Rigel	7 14·9	7 32·6	5 7 50·6	8 8·5	8 26·4	5 7 50·60

We have, then,—

	m s
Difference of times of transit	1 48·28
Difference of ÆRs (from Nautical Almanac) .	1 38·29
Difference	−9·99 sec.

Also from Nautical Almanac, we get

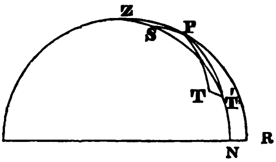
$$\text{North Polar Distance, Capella } 44^\circ\ 9'\ 21'' = \delta \ \ldots \sin \ldots\ 9{\cdot}8429911$$
$$\text{North Polar Distance, Rigel } \underline{98\ \ 22\ \ 39} = \delta' \ \ldots \sin \ldots\ 9{\cdot}1634431$$
$$+54\ \ 13\ \ 18 = \delta' - \delta \ \text{cosec}\ 10{\cdot}0908266$$

$$P = -\overset{\text{sec}}{9}\ 99 \times 15 = -149''{\cdot}85 \ .\ .\ .\ .\ 2{\cdot}1756567$$
$$\lambda = 90^\circ - 51^\circ\ 28'\ 28'' = 38^\circ\ 31'\ 32'' \ \text{cosec}\ \underline{10{\cdot}2056071}$$
$$\therefore Z = -30''{\cdot}097 \ .\ .\ .\ .\ .\ .\ .\ .\ .\ 1{\cdot}4785246$$

The value of Z being negative, the deviation is to the westward.

130. In what has been said, we have confined our attention to what takes place on that part of the meridian which stretches from the pole through the zenith down to the opposite horizon, but our formula will be equally available for observations made on that part of the meridian, between the zenith and the horizon, in which the pole is situated, if we observe that when the upper star culminates between the zenith and the pole, the arc between the two points of transit, instead of being the *difference*, is the *sum* of the polar distances, that is, the polar distance of the lower star must be considered negative; for let ZPR be the true meridian; ZSN the production of the same arc as in the last figure, traced out by the line of collimation; T the position of the star "*sub-polo*," when the transit of the upper star is observed at S; T′ the place of transit of the former; then the approximate value of ST′ is obtained by adding the polar distances. Moreover, in this

Fig. 39.



case, the difference of the right-ascensions SPT will not be the difference of the times of transit on the true meridian, but this last difference subtracted from 180°: so that in determining the value and algebraic sign of P we must subtract the supplement of the difference of the times of transit from the difference of the right-ascensions.

It appears then that the direction of the deviation will be the same as before (referred to the same part of the horizon), when the sign of Z, thus obtained, is positive, and in the other direction when the sign of Z is negative.

131. If we suppose that the *same* star is observed at both its upper and its lower transits, we shall have in the value of Z, $\delta = \delta'$; and the value of P is simply the difference of the times of transit; the value of Z, therefore, in this case becomes—

$$Z = P.\frac{\sin^2\delta}{\sin 2\delta . \sin\lambda} = \frac{P}{2}.\frac{\sin\delta}{\cos\delta . \sin\lambda} = \frac{P}{2}.\frac{\tan\delta}{\sin\lambda}.$$

If at the lower observation, the star come to the wire before its time, the deviation must be to the *westward of the pole*; and if after its time, to the eastward.

It is hardly necessary to remark that these methods are equally applicable to the altitude and azimuth instrument, when it is wished to adapt it to the particular purpose of observing the transits of bodies across the meridian.

132. We now come to the adjustments which it is necessary to attend to in the sextant before that instrument is employed for the measurement of celestial arcs. The essential points of this instrument are, (1) the perpendicularity of the mirrors at E and C to the face of the instrument; (2) their parallelism to one another when the index is at the zero-point; and (3) the parallelism of the line of collimation of the telescope to the face of the instrument. The first of these is sufficiently secured, as regards the "*index-glass*," at C, by observing whether, when the eye is applied to it so as to see the limb of the instrument and its reflected image, the two appear continuous and in the same plane: and as regards the "*horizon-glass*" at E, by observing whether an object and its reflected image exactly cover one another when seen in the ordinary manner in the object-glass from the point P, where the telescope is applied.

133. The second requisite is tested by fixing the index at the point

where an object and its reflected image are seen as one, and observing whether the index is exactly at its zero-point; if it be not, the arc indicated by the vernier should be noted as an index error, and applied to all after observations, unless it be considerable, in which case it should be reduced by slightly turning a small screw at the back of the horizon-glass; but perfect accuracy of reading should not be attempted, as meddling with any screw adjustment is sure to produce injury in the end, and a satisfactorily determined index error answers every purpose, as that error affects equally arcs of every degree of magnitude.

134. With regard to the parallelism of the line of collimation to the plane of the face of the instrument, this will be determined by observing whether the same angle is measured when the images of the two objects observed are brought into contact at parts of the field of view of the telescope, equidistant from that diameter which is parallel to the face of the instrument, since it is, strictly speaking, only on that line that the fundamental connexion between the direct and reflected rays holds good, the reflected rays which form the image at the side of the field of view falling more obliquely on the mirror than the central rays, and therefore requiring the index-glass to be shifted in order to bring the images again into contact; and this effect of oblique vision through the telescope should evidently be symmetrical on the two sides of the central line. In order to give full effect to this test, two objects should be selected which are at a very considerable angular distance from one another.

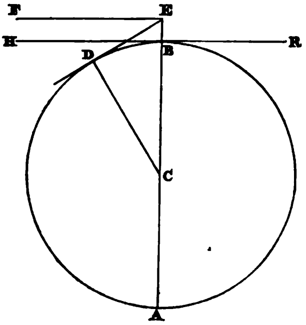
135. This instrument, having no adjustment for level, is dependent in the measurement of altitudes upon what is called an "ARTIFICIAL HORIZON," which is simply a small trough of mercury, the use of which we have already indicated in the determination of the zenith-point of the altitude and azimuth instrument. Its employment in the case of the sextant is very similar, only that both the direct ray and the reflected ray are observed at once, the former by reflexion from the two mirrors, and the latter directly through the transparent portion of the horizon-glass. Thus the angle measured is SCM, which has been shown to be equal to twice SCH which measures the altitude of the star S.

136. The use of the artificial horizon may be dispensed with when, as at sea, a natural horizon presents itself in a form which can be adapted to our purpose. The altitude of a body above the sea

horizon is not however its altitude above the rational horizon, because, in practice, the eye of the observer is always more or less raised above the surface of the water, and consequently a line drawn from the eye to the visible horizon departs more and more from one drawn in the same vertical plane parallel to the tangent plane to the surface of the earth at the place of observation, as the eye is more and more raised above that surface. In taking an altitude by this method, it is of course necessary that the point of the sea horizon, viewed through the horizon-glass, should be vertically under the body observed; this will have been effected when, by shifting the instrument steadily from side to side, the smallest obtainable angle is measured.

137. Viewing the earth as a perfect sphere, we may investigate the relation between the height of the eye and the value of the angle between these lines, which is called the "*Dip of the horizon,*" in the following manner:—

Fig. 40.



Let BDA represent a section of the earth through the place of observation B and the centre C; E the position of the eye of the observer in the prolongation of the diameter AB; then EF, drawn through E parallel to HR the section of the tangent plane at B, is the direction of the rational horizon. Draw ED to touch the circle at D and join CD; then ED is the direction of the visible horizon, and the angle FED is the "*dip.*" Now FED=ECD; if, therefore, we call the radius of the earth r; the height of the eye BE$=h$; ED$=s$, and the dip D; we have

$$AE.EB = ED^2, \quad \text{or } (2r+h)\,h = s^2;$$

but inasmuch as, in practice, AB and AE differ insensibly from one another, this may be written

$$s^2 = 2rh.$$

Again, $$\tan D = \frac{s}{r} = \sqrt{\frac{2h}{r}};$$

or, since D is necessarily very small, tan D and D are practically the same,

$$\therefore \mathrm{D}=\sqrt{\frac{2h}{r}}=\sqrt{\frac{2h}{3960\times 5280}}\left\{\begin{array}{l}\text{if } r \text{ be taken } 3960 \text{ miles, and}\\ \text{both } h \text{ and } r \text{ be estimated in feet}\end{array}\right\}$$
$$=\sqrt{\frac{2h}{3\times 1320\times 4\times 1320}}$$
$$=\frac{1}{2640}\sqrt{\frac{2h}{3}}.$$

In this formula D is estimated in circular measure; but for use, we require it in angular measure; let then D'' be the number of seconds in the dip, and we have

$$\mathrm{D}''=\frac{360\times 60\times 60.\mathrm{D}}{2\pi}=\frac{180\times 60^2\sqrt{\frac{2h}{3}}}{2640\pi}=\frac{60^2.\sqrt{6h}}{44\pi}=26.\sqrt{6h}$$

very nearly. But this formula requires to be modified, on account of terrestrial refraction, which raises the apparent above the true sea line, and which is found to diminish the above value by about its fourteenth part, so that we have, finally,

$$\mathrm{D}''=\frac{13}{14}\times 26.\sqrt{6h}=59{\cdot}14\sqrt{h},$$

in which it is to be recollected that h is the height, *in feet*, of the eye of the observer above the surface of the sea.

It is not expected that what has been here said will render unnecessary the useful hints in the manipulation of different astronomical instruments, which can alone be given *vivâ voce* with the instruments themselves to appeal to in illustration; but it is hoped that sufficient insight will have been afforded into the general principles upon which the three most useful instruments are constructed and are to be employed, to enable any one having a small amount of mathematical acquirement, to enter upon this most important and interesting branch of study in a scientific spirit; not depending upon mere rule or formula, but with a clear comprehension of the principles involved, and ready to make, in the application of those principles, whatever changes the altered circumstances of his position may demand. The object we have had in view throughout has been to enable an intelligent reader to grasp the elementary principles upon which the science of Practical Geographical Astronomy is based, and to make a fair start in actual practice, which will unfold before him, as observation after observation is made, details to be attended to, which no amount of mere reading will have previously brought under his notice.

CHAPTER VI.

ON THE TABLES OF THE NAUTICAL ALMANAC.

Description and use of the Tables.—Mode of interpolation by proportion.—Examples.—Correction for difference of Longitude.—Approximate Local Mean Time of transit.—Examples.—Recapitulation of Definitions.

138. AT the end of the Volume will be found a small collection of Tables, containing a specimen of that portion of the information given in the Nautical Almanac, which more immediately applies to the kind of observations we have illustrated in our examples: and it only remains for us to point out the mode of arrangement of these several Tables, with the uses to which they are applied, which have, however, been already indicated in the previous Examples; and then to show how the required information is to be extracted from them, to enable the reader to commence his actual practice of astronomical observation.

Tables I. II. and III. have been already referred to and made use of in some of the Examples.

Table IV. contains the apparent Æ and declination of the sun's centre, his semidiameter, the equation of time, for both apparent and mean noon at Greenwich, and also the sidereal time at mean noon, or which is the same thing, the Æ of the mean sun at that instant. The amount of change in the former of these values in one hour is also given, for the purpose of facilitating the finding of the corresponding values at any instant intermediate between any two consecutive Greenwich noons. The values given in this Table are of such constant use, that we cannot here attempt to point out every purpose to which they may be applied, further than by remarking, that they are one or more, required, in every problem, in which the position of the sun enters as an element. The Æ and declination are termed "*apparent*," because, as here given, they are affected with aberration.

The sidereal time at Greenwich mean noon is useful in a variety of problems, as it holds the same relation to the mean sun that the Æ given does to the true sun, and it therefore enables us to connect the mean time at any instant (which is, in fact, the hour-angle of the mean sun) directly with sidereal time, without the intervention of the Æ of the true sun: an inspection of the Table will show that these values are indeed only the sums of the corresponding values of the sun's Æ and the equation of time, which relation is at once evident, from our equation (art. 89.); for, calling the mean time m, and the equation of time e, we have $\alpha = m + e$, and that equation becomes

$$\sigma = m + e + \varrho;$$

but, at mean noon, $m = 0$, so that if Σ be the sidereal time at that instant, we have, for that instant,

$$\Sigma = e + \varrho.$$

In Table V. are given the mean time of the transit of the first point of Aries; together with the moon's semidiameter and horizontal parallax for every day of the month of November, 1853. The values in the first column show the mean time at the instant of the transit of the starting-point of sidereal measurements of time, just as the last column of the preceding table shows the sidereal time at the instant of transit of the starting-point (the mean sun) of mean solar measurements of time. These values may be used for the conversion of sidereal into mean solar time, in the same manner as those are employed for the inverse process: we have shown that

$$\sigma = m + e + \rho; \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1.)$$

and if $e' + \rho'$ be the Æ of the mean sun at mean noon,

$$\Sigma = e' + \rho'; \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2.)$$

hence

$$\sigma = m + \Sigma + e + \rho - (e' + \rho'). \quad . \; . \; . \; . \quad (3.)$$

In the same way, if M be the mean time at sidereal noon (that is, at the transit of the first point of Aries), since we have from (1.),

$$m = \sigma - (e + \varrho).$$

And at sidereal noon $\sigma = 24$; we get

$$\mathrm{M} = 24 - (e' - \rho'); \quad . \; . \; . \; . \; . \; . \; . \quad (4.)$$

and, hence,

$$m = \sigma + \mathrm{M} - 24 - (e + \rho) + (e' + \rho'). \quad . \; . \; . \quad (5.)$$

It will be remarked that in (4.) and (5.) the value $\sigma = 24$ has been used instead of $\sigma = 0$, merely to avoid the negative result which would otherwise arise in (4.), and which would have no very intelligible meaning: the same artifice is of constant use in all conversions of time, viz. that of rejecting 24 hours if an interval is found to exceed that time, and of introducing 24 hours if we should otherwise arrive at a negative interval by the subtraction of a greater from a less.

Now it must be observed that the expression $e+\rho-(e'+\rho')$ in (3.) and (5.) is simply the change of the Æ of the mean sun in the interval m, that is, the difference between the two values of m when viewed as a mean solar interval and as a sidereal interval; so that, if m be estimated in (3.) as a sidereal interval, and σ in (5.) as a mean solar interval, we have, calling these values m' and σ', simply,

$$\sigma=m'+\Sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.)$$

and

$$m=\sigma'+\mathrm{M}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.)$$

the 24^{h} being suppressed on the principle stated above.

Hence it appears, that if to the sidereal time at mean noon there be added the mean time at any instant, converted into a sidereal interval (by adding $9^{\mathrm{s}}{\cdot}856$ for every hour) (art. 90.), the sum will be the sidereal time at that instant; and if to the mean time at sidereal noon, there be added the sidereal time at any instant, converted into a mean solar interval (by subtracting $9^{\mathrm{s}}{\cdot}83$ for every hour) (art. 90.), the result will be the mean time at that instant.

Table VI. contains the Æ and declination of the moon for every hour of every third day in the month of November, 1853. The differences of declination for a lapse of 10^{m} of time are given to facilitate the interpolation of intermediate values: in the Almanac itself these values will be found for every day of the year; they are here given for those days of the month which are also retained in the next table.

Table VII. contains a series of lunar distances for every third day in the same month; the purpose to which this table applies has been already sufficiently dwelt upon (arts. 95. 109.).

Table VIII. contains the Æs of the moon's bright limb at its *upper* transit (that is, the transit over that portion of the meridian of Greenwich on that side of the zenith of Greenwich opposite to that in which the north pole is situated); together with the Æs of those stars which are proper to be observed with her, as pointed out in art. (95.), and also the variation of the moon's Æ in one hour of longitude (art. 95.). Moon I. indicates the limb which comes first to the meridian, Moon II. that which follows; in all cases, however, the one given is the *bright* limb (art. 95.). The remaining column, which gives the sidereal time the moon's semidiameter takes to pass the meridian, enables us to determine the time of transit of the centre from an observed transit of the bright limb.

Table IX. gives the Greenwich mean time of the eclipses of Jupiter's satellites for the same month. These values are applicable to the determination of Greenwich mean time in all parts of the earth; and hence of considerable importance in the determination of terrestrial longitudes (art. 94).

Table X. contains the Æ of Jupiter for the same month, and enables us to determine his position with respect to the meridian.

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139. As far as the Tables here given are concerned, the ordinary mode of interpolation by proportion answers almost every purpose.

On this principle, if T be the interval of time between two successive tabular values V and V′ (usually twenty-four hours), and t the interval of time from the value V to the required value v (between V and V′), then we have

$$v = \mathrm{V} + \frac{t}{\mathrm{T}}(\mathrm{V}' - \mathrm{V});$$

if the successive values in the order of time be increasing, the quantity V′ − V is positive, and *vice versâ*.

Thus, if it were required to find the Æ of the sun at $3^{\mathrm{h}}\ 45^{\mathrm{m}}$, Greenwich mean time, on November 15, 1853, we should have at mean noon, November 15 (Table IV.), $\mathrm{V} = 15^{\mathrm{h}}\ 22^{\mathrm{m}}\ 43^{\mathrm{s}}{\cdot}3$; and at mean noon, November 16, $\mathrm{V}' = 15^{\mathrm{h}}\ 26^{\mathrm{m}}\ 50^{\mathrm{s}}{\cdot}73$; $\mathrm{T} = 24$; $t = 3{\cdot}75$, and hence

$$\begin{aligned} v &= \overset{\mathrm{h}}{15}\ \overset{\mathrm{m}}{22}\ \overset{\mathrm{s}}{43}{\cdot}3 + \frac{3{\cdot}75}{24} \times 4^{\mathrm{m}}\ 7^{\mathrm{s}}{\cdot}43 \\ &= 15\ 22\ 43{\cdot}3 \\ &\quad +\qquad 38{\cdot}66 \\ &= \underline{15\ 23\ 21{\cdot}96} = \text{Æ of sun at } 3^{\mathrm{h}}\ 45^{\mathrm{m}}, \text{ Greenwich mean time.} \end{aligned}$$

The Æs at apparent noon are tabulated with the differences for every ten minutes; and we might have employed these values in finding the above result, bearing in mind that the interval t will now be affected by the equation of time, which is itself subject to a variation dependent upon the interval t. If we call the equation of time e, and its change in one hour $\frac{e'-e}{\mathrm{T}}$, which will also be found in the table ($0^s{\cdot}453$), we have, for the elapsed time from apparent noon to the time for which the Æ is required,

$$t+e+\frac{e'-e}{\mathrm{T}}\times(t+e),$$

and for the whole variation of the Æ from apparent noon to the given time,

$$\left\{t+e+\frac{(e'-e)}{\mathrm{T}}\,.\,(t+e)\right\}.\,h.$$

if h be the difference of Æ for one hour.

Using this method, we have $e=15^m\ 12^s{\cdot}95$; $\frac{e'-e}{\mathrm{T}}=-{\cdot}453$; $h=10^s{\cdot}31$; and hence the variation required will be

$$(3^h\ 45^m\ 0^s+15^m\ 12^s{\cdot}95-{\cdot}453^s\times 4{\cdot}003)10^s{\cdot}31,$$

which gives $41^s{\cdot}27$ as the value to be added to the sun's Æ at apparent noon in order to obtain the Æ at the required time; we have then by the table,

	h	m	s
Æ of sun at apparent noon	15	22	40·69
Correction for elapsed time			41·27
Æ of sun at $3^h\ 45^m$ Greenwich mean time	15	22	21·96 (as before).

Examples.

1. Find in Table IV. the sun's Æ and declination on Nov. 23, 1853, at $16^h\ 14^m\ 27^s$, Greenwich mean time, and also the apparent time at that instant.

Æ of sun . . . = $16^h\ 58^m\ 57^s{\cdot}74$
Equation of time = $0^h\ 13^m\ 19^s{\cdot}94$
Declination of sun = $20°\ 32'\ 57''{\cdot}65$

2. Find the horizontal parallax and semidiameter of the moon on Nov. 5, 1853, at $10^h\ 15^m\ 20^s$, Greenwich mean time.

From Table V. . Horizontal parallax = $59'\ 43''{\cdot}75$
Semidiameter . = $16\ 18{\cdot}5$

3. Find the lunar distance of Venus on Nov. 6, at $13^h\ 27^m\ 50^s$, and the Æ and declination of the moon at that instant.

From Table VII. Lunar distance . . = $34°\ 34'\ 15''{\cdot}40$
From Table VI. . Declination of moon = $23\ 45\ 38{\cdot}43$
Æ of moon . . = $20^h\ 23^m\ 7^s{\cdot}72$

4. Find the Greenwich mean time answering to the lunar distance of Regulus, 22° 20′ 17″, on the 21st of November, 1853.

From Table VII. Greenwich mean time = $8^h\ 57^m\ 55^s{\cdot}94$

5. Find the Greenwich mean time answering to the lunar distance of α Pegasi, 38° 7′ 41″, on the 12th of November 1853.

Greenwich mean time $= 16^h\ 3^m\ 46^s{\cdot}44$

6. At a certain place the true distance of α Arietis from the moon's centre was found to be 93° 5′ 7″ at $14^h\ 27^m\ 18^s$, local mean time, 6th November 1853; find the longitude of the place.

Longitude $= 0^h\ 6^m\ 1^s{\cdot}38$ W.

140. If the place at which an observation is made be not on the meridian of Greenwich, allowance must, of course, be made for the difference of longitude in extracting the required values from the tables, adding this difference to the local time to obtain the corresponding Greenwich time, when the place is to the westward of Greenwich, and *vice versâ*. Thus, let it be required to find the Æ of the moon at $7^h\ 14^m\ 27^s$, local mean time, November 7, 1853, at a place in longitude 17° 16′ 36″ E. This difference of longitude is equivalent to $1^h\ 9^m\ 6^s{\cdot}2$, which gives Greenwich mean time $= 6^h\ 5^m\ 20^s{\cdot}8$, and by Table VI., difference of moon's Æ for $1^m = 8^s{\cdot}41$;

∴ Difference Æ for $5^m\ 20^s{\cdot}8 = 8^s{\cdot}41 \times 5{\cdot}347 =$ $44^s{\cdot}97$
Æ of moon at $6^h\ 0^m\ 0^s$ . . . . . . $= 21^h\ 3^m\ 25^s{\cdot}67$
∴ Æ of moon at $7^h\ 14^m\ 27^s$, local mean time $= 21^h\ 4^m\ 10^s{\cdot}64$

141. In cases where the longitude is required to be found by the assistance of the variable data given in the Almanac, an approximate Greenwich time, and therefore an approximate longitude can always be obtained sufficiently accurate for the purpose of reference to the Almanac, by first making use of the tabulated value nearest to that which we should employ if the longitude were known; with this approximate longitude a more accurate value can afterwards be found from the Tables. In most cases, however, the longitude is already known with sufficient accuracy for our purpose, either from our approximate position being known with reference to some known place, not far off, or "*by account*," that is, by a general estimate of the direction and distance travelled day after day. This last is the mode usually adopted at sea; the direction and rate of

sailing being entered hour by hour in the ship's log, which thus becomes a record of the space passed over in longitude and in latitude since the departure of the ship from some known headland.

142. For example, it frequently happens that, in determining the longitude from a lunar distance at sea, the state of the weather has prevented the altitudes of the moon and star being observed, and the longitude by account is not to be much depended upon; in this case the Æ and declination of the moon which are required to be taken from the Almanac in order to find the altitude at the time of observation, must be obtained by means of the approximate Greenwich mean time derived from the apparent lunar distance.

Example.

Find the Æ of the moon on November 4, when the observed lunar distance of Fomalhaut was 59° 2′ 54″.

To find the Greenwich mean time approximately, we have (Table VII.),

$$\text{Greenwich mean time} = \overset{\text{h}}{12} + \overset{\text{h}}{3} \times \frac{60^\circ\ 15'\ 34^{\text{s}} - 59^\circ\ \ 2'\ 54''}{60^\circ\ 15'\ 34^{\text{s}} - 58^\circ\ 31'\ 36''}$$

$$= \overset{\text{h}}{12} + \overset{\text{h}}{3} \times \frac{4360}{6238} = \overset{\text{h}}{12} + \overset{\text{h}}{2{\cdot}1} = \overset{\text{h}}{14}\ \overset{\text{m}}{6}\ \overset{\text{s}}{0}$$

Æ of moon, Nov. 4, $15^{\text{h}}\ 0^{\text{m}}\ 0^{\text{s}}$ . . . . . . $\overset{\text{h}}{18}\ \overset{\text{m}}{21}\ \overset{\text{s}}{32{\cdot}99}$

Difference for $3^{\text{h}} = 8^{\text{m}}\ 4^{\text{s}}{\cdot}75$;

$\therefore$ Difference for $54^{\text{m}} = \frac{3}{10} \times 484^{\text{s}}{\cdot}75$ . . . . $2\ 25{\cdot}42$

Æ of moon at $14^{\text{h}}\ 6^{\text{m}}$, Greenwich mean time $= 18\ 19\ \ 7{\cdot}57$

143. In making transit observations, it is convenient to find the approximate mean local time at which a body may be expected to cross the meridian, and this can readily be derived from the Æ of the body combined with the mean time of transit of the first point of Aries, which is, in fact, the sidereal noon. For by converting the Æ of the body into a mean solar interval, and adding it to the mean time answering to the sidereal noon, we have at once the mean time of transit. If the place of observation be situated upon a meridian whose west longitude is L, we may first determine the mean time of transit of the first point of Aries for that meridian, from that given for Greenwich, in the Table, which we can do by interpolating the value required between those of the previous and

following dates at Greenwich. This value, according to our former notation, will be

$$v=V+(V'-V)\frac{L}{24};$$

and adding it to the Æ of the body, converted into a mean solar interval, we shall obtain the mean time of the transit of the body.

We may also obtain the required mean time of transit by subtracting the Æ of the mean sun at the local mean noon (that is, the sidereal time at mean noon) from the Æ of the body, and then converting this result into a mean time interval. Thus, let it be required to find the local mean time of transit of Aldebaran on 10th November 1853, in longitude 27° 14′ 30″ E.

By first method.

$-27° \, 14' \, 30'' = -2^h \, 16^m \, 12^s{\cdot}5 = -2^h{\cdot}27$

	h	m	s	
From Table V. V =	8	40	21·16	
V′ =	8	36	25·25	
V′ − V = −		3	55·91*	= −235·91 sec. (*Mean*).
V =	8	40	21·16	
$(-235{\cdot}91)\times\left(-\frac{2{\cdot}27}{24}\right)=$			+22·31	
∴ v =	8	40	43·47	= Local mean time of transit of ♈.
Æ Aldebaran = 4 27 29				
Retardation = 43·82				
	4	26	45·18	
	13	7	28·65	Local mean time of transit of Aldebaran.

By second method.

	h	m	s	
Sidereal time at Greenwich mean noon, 9th......	15	14	16·80	
Sidereal time at Greenwich mean noon, 10th......	15	18	13·36	
		3	56·56*	= 236·56 sec. (*Sid.*)
Sidereal time at Greenwich mean noon, 10th ...	15	18	13·36	
$236{\cdot}56\times\frac{2{\cdot}27}{24}=$			22·38	
Sidereal time at local mean noon	15	17	50·98	
Æ Aldebaran..	4	27	29	
	13	9	38·02	
Retardation for 13ʰ·16		2	9·36	
Local mean time of transit of Aldebaran	13	7	28·66	(as before).

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* This will always be the difference, because the result of the subtraction must be the excess of a mean solar over a sidereal day (art. 82.).

Supposing the body, whose time of transit is required, to be the moon, we can obtain the Æ of the bright limb at the Greenwich transit, from Table VIII., and add or subtract the amount of variation of Æ for the difference of longitude, according as the place is to the westward or eastward of Greenwich, in order to find the Æ of the bright limb at the transit over the meridian of the place; and then proceed as before.

Let it be required to find the local mean time of transit of the moon's bright limb at a place in longitude $4^{h}\ 17^{m}\ 39^{s}$ E. on November 27, 1853; we have, by Table VIII.,

	h	m	s
Var. of Æ for $4^{h}\cdot294 = 4\cdot294 \times 140^{s}\cdot67 = 604^{s}\cdot02 =$	0	10	4·02
Æ Moon's bright limb at Greenwich transit =	14	7	11·26
Æ Moon's bright limb at required transit =	13	57	7·24
and, proceeding as before, we get the Local mean time of transit =	21	29	2·51

Should the mean time of transit of the moon's centre be required, the sidereal time of transit of her semidiameter, derived from Table VIII. (converted into a mean solar interval if great accuracy be required), added to, or subtracted from the above result, according as the bright limb is to the westward or eastward of the centre, will give the value required.

Miscellanous Examples.

1. Find the local mean time of transit of the moon's bright limb at the Cape of Good Hope, in longitude $1^{h}\ 13^{m}\ 55^{s}$ E., on November 7, 1853. Local Mean Time $=5^{h}\ 51^{m}\ 52^{s}\cdot35$.

2. Find the local mean time of transit of Rigel on November 22, 1853, at Madras, in longitude $5^{h}\ 21^{m}\ 4^{s}$ E. L.M.T. $=13^{h}\ 0^{m}\ 37^{s}\cdot1$.

3. Find the horizontal parallax of the moon at St. Helena, in longitude $0^{h}\ 22^{m}\ 50^{s}$ E., at $9^{h}\ 27^{m}\ 4^{s}$, local mean time.
Horizontal Parallax $=56'\ 55''\cdot23$.

4. Find the equation of time at mean noon at Valparaiso, in longitude 71° 35′ W., on November 17, 1853. E.T. $=14^{m}\ 47^{s}\cdot76$.

5. Find the Æ of the sun, November 23, 1853, at $7^{h}\ 14^{m}\ 21^{s}$, local mean time, at Hobarton, in longitude 147° 26′ E.
Æ$=15^{h}\ 55^{m}\ 37^{s}\cdot03$.

6. The longitude of a ship at sea by account was $3^{h}\ 17^{m}\ 10^{s}$ W.; find the Æ of the sun at apparent noon, November 28, 1853.
Æ$=16^{h}\ 18^{m}\ 5^{s}\cdot8$.

7. The true distance of the moon's centre from α Aquilæ was found to be 55° 10′ 20″, in November 1853, at $10^h\ 17^m\ 24^s$, local mean time; find the mean time at which Capella will be on the meridian of the place where the observation was made.

Local Mean Time $= 13^h\ 48^m\ 6^s\cdot41$.

8. On November 12, 1853, the transit of the moon's bright limb was observed, by mean time chronometer $35^m\ 4^s$ after that of δ Piscium; find the longitude of the place of observation.

Longitude $= 0^h\ 42^m\ 5^s\cdot76$ W.

9. On November 22, 1853, the transit of the moon's bright limb was observed by sidereal clock $18^m\ 10^s\cdot3$ before that of Regulus (α Leonis); find the longitude of the place of observation.

Longitude $= 2^h\ 45^m\ 36^s$ W.

10. The altitude of a comet being observed 17° 14′ 20″ at $6^h\ 43^m\ 10^s$, local mean time, November 14, 1853, when its azimuth was N. 77° 44′ 10″ W.; find its Æ and declination, the latitude and longitude of the place of observation being 58° 22′ 47″ and $1^h\ 46^m\ 55^s$ E.

Æ $= 16^h\ 42^m\ 38^s\cdot92$.

Dec. $= 29^h\ 14^m\ 3^s$ N.

11. The re-appearance of the first satellite of Jupiter was observed November 11, 1853, $4^m\ 10^s$ (sidereal) before the transit of β Scorpii, at an observed altitude of 64° 17′ 10″; find the latitude and longitude of the place.

Latitude = 6° 18′ 54″ N.

Longitude $= 8^h\ 5^m\ 16^s\cdot08$ E.

12. The re-appearance of the second satellite of Jupiter was observed on November 20, 1853, at $21^h\ 24^m\ 50^s$, sidereal time; find the longitude of the place.

Longitude $= 16^h\ 3^m\ 33^s\cdot13$.

13. At $10^h\ 47^m\ 13^s$, sidereal time, November 24, 1853, the true lunar distance of Spica Virginis was found to be 33° 10′ 40″; find the longitude of the place.

Longitude $= 0^h\ 55^m\ 40^s\cdot53$ W.

14. The altitude of the sun's lower limb at apparent noon of the place in the last question, on November 25, 1853, was observed to be 31° 17′ 54″; find the latitude of the place.

Lat. = 37° 38′ 19″ N.

15. The transit of the sun's centre was observed at $14^h\ 27^m\ 32^s\cdot4$, Greenwich mean time, November 17, 1853; find the longitude of the place of observation.

Longitude $= 14^h\ 12^m\ 49^s\cdot45$.

Summary of Definitions of the Astronomical Terms explained and used in the Text; alphabetically arranged.

ALTITUDE.—The *True* Altitude of a celestial body is the angle which the straight line drawn from the body to the centre of the earth makes with the plane of the rational horizon (art. 24.).

The *Apparent* Altitude of a body is the angle which the direction of the ray of light, proceeding from the body, makes with the plane of the horizon, at the moment when it enters the eye of the observer.

APPARENT TIME.—See Time, Apparent.

ARIES, FIRST POINT OF.—See First Point of Aries.

AUTUMNAL EQUINOX.—See Equinox.

AZIMUTH.—The Azimuth of a body is the angle of inclination of that vertical plane which passes through the body to another vertical plane which passes through some fixed point in the horizon. If nothing to the contrary be expressed, it is understood to be the angle of inclination of the former plane to the plane of the meridian, measured on that side on which the pole is situated (art. 25.).

CULMINATION.—A body is said to culminate at the instant of its transit across the meridian (art. 57).

DAY.—See Time.

DECLINATION.—The Declination of a celestial body is the angle which the straight line drawn from the body to the centre of the earth makes with the plane of the equator. It is measured by the arc of a secondary to the equator passing through the body, intercepted between the body and the equator (art. 61.).

EAST.—The East point of the horizon is the point where the prime vertical meets it, on that side of the meridian on which the celestial bodies rise (art. 58.).

ECLIPTIC.—The Ecliptic is that great circle of the heavens which the sun traces out in the course of a sidereal year (art. 63.).

EQUATION OF TIME.—See Time, Equation of.

EQUATOR.—The Celestial Equator is that great circle of the heavens which is traced out by the plane of the terrestrial equator (art. 58.).

EQUINOX.—The Vernal Equinox is that point in the equator over which the centre of the sun crosses in passing from the Southern to the Northern Hemisphere (art. 66.).

The **AUTUMNAL EQUINOX** is that point in the equator over which the centre of the sun crosses in passing from the Northern to the Southern Hemisphere (art. 65.).

The Equinoxes are the points in which the ecliptic cuts the equator.

First Point of Aries.—The First Point of Aries is the Vernal Equinox, and it is the point from which the Right Ascensions of all bodies are measured (art. 65.).

Horizon.—The Rational Horizon of a place on the earth is the great circle of the heavens which is traced out by a plane drawn through the centre of the earth, perpendicular to that radius of the earth which passes through the place (art. 57.).

The Sensible Horizon of a place is that plane which is drawn through the place tangential to the surface of the earth.

Horizontal Parallax.—See Parallax.

Hour-Angle.—The Hour-Angle of a celestial body is the spherical angle contained at the celestial pole by the meridian, and that secondary to the equator which passes through the body (art. 96.). It is measured by the arc of the equator intercepted between the meridian and that secondary (art. 96.).

Latitude.—The Latitude of a celestial body is the angle which the straight line drawn from the body to the centre of the earth makes with the plane of the ecliptic (art. 71.). It is measured by the arc of a secondary to the ecliptic passing through the body intercepted between the body and the ecliptic (art. 71.).

The Latitude of a place on the earth is the arc of a secondary to the earth's equator passing through the place, intercepted between the place and the equator.

Longitude.—The Longitude of a celestial body is the angle which the straight line joining the centre of the earth and the first point of Aries makes with the plane of that secondary to the ecliptic which passes through the body (art. 71.). It is measured by the arc of the ecliptic which is intercepted between the first point of Aries and the secondary (art. 71.).

The Longitude of a place on the earth is an arc of the equator intercepted between the meridian of that place and the meridian of some other place which is established as the meridian of reference for terrestrial longitudes.

Mean Time.—See Time, Mean.

North.—The North Point of the horizon is the point in which the meridian cuts the horizon on that side of the prime vertical on which the north pole is situated (art. 58.).

Obliquity of the Ecliptic.—The Obliquity of the Ecliptic is the inclination of the plane of the ecliptic to that of the equator (art. 63.). It is measured by the declination of the sun at each solstice.

Parallax.—The Parallax of a body is the amount of change in its apparent position relatively to the zenith of the observer, as seen from the surface and from the centre of the earth: its amount is measured by the angle subtended at the body by that radius of the earth which passes through the place of observation (art. 40.). The Horizontal Parallax of a body is the angle subtended at the body by the same radius of the earth when the body is in the horizon of the observer (art. 41.).

POLES.—The Celestial Poles are the points in the heavens round which the stars appear to revolve; they are the points in which the axis of the earth would, if produced, meet the imaginary sphere upon which we conceive the celestial bodies to be projected (art. 28.).

PRECESSION.—The Precession of the Equinoxes is the change which takes place in the position of the equinoxes, owing to a gradual alteration in the direction of the earth's axis, and, consequently, in the position of the celestial poles (art. 66.).

PRIME VERTICAL.—The Prime Vertical is that great circle of the heavens the plane of which passes through the zenith, and is perpendicular to the plane of the meridian (art. 58.).

REFRACTION.—The Refraction of a heavenly body is the change of direction which a ray of light, proceeding from the body to the eye of the observer, undergoes in passing through the atmosphere (art. 48.).

RIGHT-ASCENSION.—The Right-Ascension of a body is the arc of the celestial equator between the vernal equinox or first point of Aries and the point where a secondary to the equator passing through the body meets the equator. This arc is measured in the direction of the apparent annual motion of the sun, that is, from west to east (art. 70.).

SIDEREAL TIME.—See Time, Sidereal.

SOLSTICES.—The Solstices are the points in the ecliptic, 90° distant from the equinoxes, where the apparent annual motion of the sun being parallel to the equator, the change in its declination in any small interval is 0 (art. 67.).

SOUTH.—The South point of the horizon is the point where the meridian meets the horizon on that side of the prime vertical most remote from the north pole of the heavens (art. 58.).

TIME.—Apparent Time is the number of hours, minutes and seconds which have elapsed at any particular instant since the centre of the sun was on the meridian of the place of observation; twenty-four of such hours, or a *Solar Day* (art. 81.), being the interval between the two consecutive transits of the sun's centre over the same meridian, on the day of observation (art. 86.).

The **Mean Time** at any instant is the number of hours, minutes and seconds which have then elapsed since a certain point in the equator (which having at or about noon on the 24th of December the same Æ as the sun, moves uniformly in the equator, so as to have again the same Æ when the sun has again that Æ) was last upon the meridian of the place of observation; twenty-four of such hours, or a *Mean Solar Day*, being the interval between two consecutive transits of that point over the same meridian (art. 83.).

The **Sidereal Time** at any instant is the number of hours, minutes and seconds which have then elapsed since the first point of Aries was upon the meridian of the place of observation; twenty-four of such hours, or a *Sidereal Day*, being the interval between two consecutive transits of the first point of Aries over the same meridian (art. 76.).

The **Equation of Time**, at any instant, is the difference between the apparent time and the mean time at that instant (art. 83.).

Transit.—The Transit of a heavenly body is the act of its apparent passage across any great circle of the celestial sphere: when nothing to the contrary is expressed, this term is applied to that passage across the meridian which is nearest to the point of the horizon most remote from the elevated pole of the heavens.

Vernal Equinox.—See Equinox, Vernal.

Vertical.—Vertical circles are those great circles which are secondaries to the rational horizon: the prime vertical is that vertical circle which is also a secondary to the meridian (art. 58.).

West.—The West Point of the horizon is that one of the points where the prime vertical cuts the horizon, which is on that side of the meridian towards which all the heavenly bodies appear to move as they approach the horizon (art. 58.).

Year.—A Sidereal year is the interval of time which elapses between the instant at which the sun has the same Æ as any particular star, and the instant at which it has next again the same Æ (art. 81.).

A Tropical year is the interval of time which elapses between the instant of the sun's passing one of the equinoctial points, and his next again passing that point.

Zenith.—The Zenith of a place is that point in the heavens which is the pole of the horizon (art. 25.).

Zodiac.—The Zodiac is a belt of the celestial sphere of a certain, the same, number of degrees on both sides of the ecliptic. It is divided into the twelve signs called "the signs of the Zodiac." This term is almost obsolete.

TABLES.

TABLE I.—Astronomical Mean Refractions for the Temperature 50° Fahrenheit, and the Barometric Pressure 30 inches of Mercury.

(From a paper by Sir James Ivory in the Philosophical Transactions for 1838.)

Zenith Distance.	Mean Refraction.	Zenith Distance.	Mean Refraction.	Zenith Distance.	Mean Refraction.
1° 0′	0′ 1″	73° 30″	3′ 15″	85° 10′	10° 11″
10 0	0 10	74 0	3 21	85 20	10 28
20 0	0 21	74 30	3 28	85 30	10 46
30 0	0 34	75 0	3 35	85 40	11 6
35 0	0 41	75 30	3 42	85 50	11 26
40 0	0 49	76 0	3 50	86 0	11 47
45 0	0 58	76 30	3 59	86 10	12 10
50 0	1 10	77 0	4 8	86 20	12 34
52 0	1 15	77 30	4 18	86 30	13 0
54 0	1 20	78 0	4 28	86 40	13 27
56 0	1 26	78 30	4 40	86 50	13 56
58 0	1 33	79 0	4 52	87 0	14 27
60 0	1 41	79 20	5 1	87 10	15 0
61 0	1 45	79 40	5 10	87 20	15 35
62 0	1 49	80 0	5 20	87 30	16 12
63 0	1 54	80 20	5 31	87 40	16 52
64 0	1 59	80 40	5 42	87 50	17 35
65 0	2 5	81 0	5 54	88 0	18 21
66 0	2 10	81 20	6 7	88 10	19 11
67 0	2 17	81 40	6 20	88 20	20 5
68 0	2 24	82 0	6 35	88 30	21 3
69 0	2 31	82 20	6 50	88 40	22 5
69 30	2 35	82 40	7 7	88 50	23 13
70 0	2 39	83 0	7 25	89 0	24 27
70 30	2 44	83 20	7 45	89 10	25 47
71 0	2 48	83 40	8 7	89 20	27 14
71 30	2 53	84 0	8 30	89 30	28 50
72 0	2 58	84 20	8 55	89 40	30 33
72 30	3 3	84 40	9 23	89 50	32 15
73 0	3 9	85 0	9 54	90 0	34 32

TABLE II.—Showing the factor to be applied to the values in Table I. to correct them for variations of temperature and pressure.

(Derived from the same source as Table I.)

Temperature.	Pressure.											
	28·5	28·7	28·9	29·1	29·3	29·5	29·7	29·9	30·1	30·3	30·5	30·7
20°	1·016	1·023	1·031	1·038	1·045	1·052	1·059	1·066	1·073	1·080	1·087	1·095
30	0·993	1·000	1·007	1·014	1·021	1·028	1·035	1·042	1·049	1·056	1·063	1·070
40	0·971	0·978	0·985	0·992	0·999	1·006	1·012	1·019	1·026	1·033	1·039	1·046
50	0·950	0·957	0·963	0·970	0·977	0·983	0·990	0·997	1·003	1·010	1·017	1·023
60	0·930	0·936	0·943	0·949	0·956	0·962	0·969	0·975	0·982	0·988	0·995	1·002
70	0·910	0·917	0·923	0·929	0·936	0·942	0·949	0·955	0·961	0·968	0·974	0·980
80	0·891	0·898	0·904	0·910	0·917	0·923	0·929	0·935	0·942	0·948	0·954	0·960

TABLE III.

Mean places of some of the principal Fixed Stars which are visible in the Latitude of London. January 1, 1853.

Name of Star.	Æ.			Declination.		
	h	m	s	°	′	″
α Andromedæ . . .	0	0	48	28	16	44 N.
γ Pegasi (*Algenib*)	0	5	40	14	21	58 N.
α Cassiopeæ	0	32	12	55	43	50 N.
β Ceti.	0	36	12	18	47	40 s.
α Ursæ Minoris (*Polaris*)	1	5	54	88	31	33 N.
α Arietis	1	58	54	22	45	54 N.
γ Ceti	2	35	41	2	36	48 N.
α Ceti	2	54	36	3	30	36 N.
α Persei.	3	13	51	49	20	0 N.
α Tauri (*Aldebaran*)	4	27	29	16	12	34 N.
α Aurigæ (*Capella*)	5	5	50	45	50	33 N.
β Orionis (*Rigel*) .	5	7	28	8	22	32 s.
β Tauri	5	17	0	28	28	41 N.
δ Orionis	5	24	30	0	24	44 s.
ι Orionis	5	28	45	1	18	0 s.
α Orionis (*Betelgeux*)	5	47	13	7	22	30 N.
α Canis Majoris (*Sirius*)	6	38	40	16	31	6 s.
α Geminorum (*Castor*)	7	25	13	32	12	21 N.
α Canis Minoris (*Procyon*)	7	31	36	5	35	53 N.
β Geminorum (*Pollux*)	7	36	19	28	22	36 N.
α Hydræ	9	20	22	8	1	26 s.
α Leonis (*Regulus*)	10	0	32	12	41	1 N.
α Ursæ Majoris (*Dubhe*)	10	54	37	62	32	36 N.
δ Leonis.	11	6	17	21	19	42 N.
β Leonis	11	41	33	15	23	37 N.
γ Ursæ Majoris . .	11	46	5	54	30	43 N.
α Virginis (*Spica*).	13	17	27	10	23	33 s.
η Ursæ Majoris . . .	13	41	45	50	2	54 N.
α Boōtis (*Arcturus*)	14	8	57	19	56	59 N.
α Libræ	14	42	45	15	25	40 s.
β Ursæ Minoris . .	14	51	11	74	45	22 N.
β Libræ	15	9	6	8	50	14 s.
α Coronæ Borealis	15	28	28	27	12	44 N.
α Serpentis	15	37	2	6	53	29 N.
β Scorpii	15	56	54	19	23	56 s.
δ Ophiuchi	16	6	39	3	18	44 s.
α Scorpii (*Antares*)	16	20	24	26	6	4 s.
α Herculis	17	7	57	14	33	41 N.
β Draconis.	17	27	7	52	24	43 N.
α Ophiuchi.	17	28	7	12	40	15 N.
γ Draconis.	17	53	12	51	30	29 N.
α Lyræ (*Vega*) . . .	18	31	58	38	38	58 N.
β Lyræ	18	44	39	33	11	41 N.
δ Aquilæ	19	18	5	2	49	32 N.
γ Aquilæ	19	39	16	10	15	30 N.
α Aquilæ (*Altair*).	19	43	37	8	29	1 N.
β Aquilæ	19	48	5	6	2	35 N.
α Capricorni.	20	9	54	12	59	49 s.
α Cycni	20	36	25	44	45	26 N.
α Cephei	21	15	4	61	57	50 N.
β Aquarii.	21	23	49	6	12	55 s.
β Cephei	21	26	45	69	54	57 N.
α Aquarii.	21	58	14	1	1	56 s.
α Gruis	21	58	57	47	40	12 s.
α Piscis Australis (*Fomalhaut*)	22	49	31	30	23	59 s.
α Pegasi (*Markab*)	22	57	26	14	24	55 N.
γ Cephei	23	33	21	76	48	44 N.

TABLE IV.—NOVEMBER, 1853.

AT APPARENT NOON.							
Day of the Week.	Day of the Month.	THE SUN'S				Equation of Time *to be subtracted from Apparent Time.*	Diff. for 1 hour.
		Apparent Right Ascension.	Diff. for 1 hour.	Apparent Declination.	Diff. for 1 hour.		
		h m s	s	° ′ ″	″	m s	s
Tuesday . .	1	14 26 23·94	9·828	S. 14 30 23·7	47·76	16 17·74	0·029
Wednesday	2	14 30 19·80	9·861	14 49 30·0	47·16	16 18·44	0·005
Thursday .	3	14 34 16·47	9·895	15 8 21·8	46·54	16 18·33	0·039
Friday . . .	4	14 38 13·95	9·929	15 26 58·7	45·90	16 17·40	0·072
Saturday . .	5	14 42 12·24	9·963	15 45 20·2	45·24	16 15·67	0·106
Sunday . . .	6	14 46 11·35	9·997	16 3 26·0	44·57	16 13·13	0·140
Monday . .	7	14 50 11·28	10·032	16 21 15·6	43·87	16 9·76	0·175
Tuesday . .	8	14 54 12·04	10·066	16 38 48·5	43·16	16 5·56	0·209
Wednesday	9	14 58 13·62	10·101	16 56 4·4	42·44	16 0·55	0·243
Thursday .	10	15 2 16·04	10·135	17 13 3·0	41·69	15 54·71	0·278
Friday . . .	11	15 6 19·29	10·171	17 29 43·6	40·94	15 48·03	0·313
Saturday . .	12	15 10 23·39	10·205	17 46 6·1	40·16	15 40·51	0·348
Sunday . . .	13	15 14 28·31	10·240	18 2 10·0	39·38	15 32·17	0·383
Monday . .	14	15 18 34·08	10·275	18 17 55·0	38·57	15 22·98	0·418
Tuesday . .	15	15 22 40·69	10·310	18 33 20·6	37·75	15 12·95	0·453
Wednesday	16	15 26 48·14	10·346	18 48 26·5	36·91	15 2·09	0·488
Thursday .	17	15 30 56·44	10·381	19 3 12·4	36·06	14 50·38	0·523
Friday . . .	18	15 35 5·58	10·415	19 17 37·8	35·19	14 37·83	0·557
Saturday . .	19	15 39 15·55	10·450	19 31 42·4	34·31	14 24·46	0·593
Sunday . . .	20	15 43 26·36	10·485	19 45 25·9	33·42	14 10·24	0·627
Monday . .	21	15 47 38·00	10·519	19 58 48·0	32·50	13 55·19	0·661
Tuesday . .	22	15 51 50·46	10·553	20 11 48·1	31·58	13 39·33	0·694
Wednesday	23	15 56 3·73	10·586	20 24 26·0	30·64	13 22·67	0·728
Thursday .	24	16 0 17·80	10·619	20 36 41·4	29·68	13 5·20	0·760
Friday . . .	25	16 4 32·66	10·651	20 48 33·8	28·72	12 46·95	0·793
Saturday . .	26	16 8 48·29	10·682	21 0 3·0	27·73	12 27·93	0·823
Sunday . . .	27	16 13 4·66	10·713	21 11 8·6	26·73	12 8·17	0·854
Monday . .	28	16 17 21·78	10·743	21 21 50·2	25·72	11 47·67	0·884
Tuesday . .	29	16 21 39·60	10·772	21 32 7·5	24·70	11 26·46	0·913
Wednesday	30	16 25 58·12	10·799	21 42 0·3	23·66	11 4·56	0·940
Thursday .	31	16 30 17·30		S. 21 51 28·1		10 42·00	

TABLE IV. (Continued.)—NOVEMBER, 1853.

AT MEAN NOON.						
Day of the Week.	Day of the Month.	THE SUN'S			Equation of Time *to be added to Mean Time.*	Sidereal Time.
		Apparent Right Ascension.	Apparent Declination.	Semidiameter.		
		h m s	° ′ ″	′ ″	m s	h m s
Tuesday . .	1	14 26 26·61	S. 14 30 36·7	16 9·8	16 17·75	14 42 44·36
Wednesday	2	14 30 22·48	14 49 42·9	16 10·1	16 18·44	14 46 40·92
Thursday .	3	14 34 19·15	15 8 34·5	16 10·3	16 18·32	14 50 37·47
Friday . . .	4	14 38 16·64	15 27 11·2	16 10·5	16 17·38	14 54 34·03
Saturday. .	5	14 42 14·94	15 45 32·5	16 10·8	16 15·64	14 58 30·58
Sunday. . .	6	14 46 14·05	16 3 38·0	16 11·0	16 13·09	15 2 27·14
Monday . .	7	14 50 13·98	16 21 27·4	16 11·3	16 9·71	15 6 23·69
Tuesday . .	8	14 54 14·74	16 39 0·1	16 11·6	16 5·51	15 10 20·25
Wednesday	9	14 58 16·32	16 56 15·7	16 11·8	16 0·48	15 14 16·80
Thursday .	10	15 2 18·73	17 13 14·0	16 12·0	15 54·63	15 18 13·36
Friday . . .	11	15 6 21·97	17 29 54·4	16 12·2	15 47·94	15 22 9·92
Saturday. .	12	15 10 26·05	17 46 16·6	16 12·4	15 40·42	15 26 6·47
Sunday. . .	13	15 14 30·96	18 2 20·2	16 12·7	15 32·07	15 30 3·03
Monday . .	14	15 18 36·71	18 18 4·9	16 12·9	15 22·87	15 33 59·59
Tuesday . .	15	15 22 43·30	18 33 30·2	16 13·1	15 12·84	15 37 56·14
Wednesday	16	15 26 50·73	18 48 35·8	16 13·3	15 1·96	15 41 52·70
Thursday .	17	15 30 59·01	19 3 21·3	16 13·5	14 50·25	15 45 49·26
Friday . . .	18	15 35 8·12	19 17 46·4	16 13·7	14 37·69	15 49 45·81
Saturday. .	19	15 39 18·06	19 31 50·7	16 13·9	14 24·31	15 53 42·37
Sunday. . .	20	15 43 28·83	19 45 33·8	16 14·1	14 10·09	15 57 38·92
Monday . .	21	15 47 40·44	19 58 55·5	16 14·3	13 55·04	16 1 35·48
Tuesday . .	22	15 51 52·86	20 11 55·2	16 14·4	13 39·18	16 5 32·04
Wednesday	23	15 56 6·09	20 24 32·8	16 14·6	13 22·51	16 9 28·60
Thursday .	24	16 0 20·12	20 36 47·9	16 14·8	13 5·04	16 13 25·15
Friday . . .	25	16 4 34·93	20 48 40·0	16 15·0	12 46·78	16 17 21·71
Saturday. .	26	16 8 50·51	21 0 8·8	16 15·1	12 27·76	16 21 18·27
Sunday. . .	27	16 13 6·83	21 11 14·0	16 15·3	12 8·00	16 25 14·82
Monday . .	28	16 17 23·89	21 21 55·2	16 15·5	11 47·49	16 29 11·38
Tuesday . .	29	16 21 41·66	21 32 12·2	16 15·6	11 26·28	16 33 7·94
Wednesday	30	16 26 0·11	21 42 4·7	16 15·8	11 4·38	16 37 4·50
Thursday .	31	16 30 19·23	S. 21 51 32·1	16 15·9	10 41·83	16 41 1·06

TABLE V.—NOVEMBER, 1853.

MEAN TIME.					
Day of the Month.	Transit of the First Point of Aries.	THE MOON'S			
		Semidiameter.		Horizontal Parallax.	
		Noon.	Midnight.	Noon.	Midnight.
	h m s	′ ″	′ ″	′ ″	′ ″
1	9 15 44 35	16 35·5	16 38·0	60 46·0	60 55·0
2	9 11 48·44	16 39·3	16 39·3	60 59·7	60 59·7
3	9 7 52·53	16 38·1	16 35·7	60 55·4	60 46·8
4	9 3 56·62	16 32·3	16 28·1	60 34·6	60 19·2
5	9 0 0·71	16 23·2	16 17·7	60 1·1	59 40·8
6	8 56 4·80	16 11·7	16 5·6	59 19·1	58 56·5
7	8 52 8·89	15 59·3	15 52·9	58 33·5	58 10·4
8	8 48 12·98	15 46·7	15 40·7	57 47·6	57 25·4
9	8 44 17·07	15 34·9	15 29·3	57 3·9	56 43·5
10	8 40 21·16	15 24·1	15 19·1	56 24·3	56 6·3
11	8 36 25·25	15 14·5	15 10·3	55 49·6	55 34·1
12	8 32 29·34	15 6·4	15 2·7	55 19·7	55 6·4
13	8 28 33·43	14 59·4	14 56·5	54 54·3	54 43·3
14	8 24 37·52	14 53·8	14 51·5	54 33·5	54 24·9
15	8 20 41·61	14 49·4	14 47·7	54 17·4	54 11·0
16	8 16 45·70	14 46·3	14 45·2	54 5·9	54 2·1
17	8 12 49·79	14 44·6	14 44·2	53 59·7	53 58·7
18	8 8 53·87	14 44·3	14 45·0	53 59·2	54 1·2
19	8 4 57·96	14 46·0	14 47·5	54 4·9	54 10·4
20	8 1 2·05	14 49·5	14 52·1	54 17·8	54 27·3
21	7 57 6·14	14 55·3	14 59·1	54 38·9	54 52·8
22	7 53 10·23	15 3·4	15 8·4	55 9·0	55 27·3
23	7 49 14·32	15 14·0	15 20·2	55 47·9	56 10·5
24	7 45 18·41	15 27·0	15 34·3	56 35·2	57 1·7
25	7 41 22·50	15 41·8	15 49·7	57 29·6	57 58·5
26	7 37 26·59	15 57·8	16 5·8	58 27·8	58 57·2
27	7 33 30·68	16 13·6	16 21·0	59 26·0	59 53·2
28	7 29 34·76	16 27·8	16 33·9	60 18·2	60 40·2
29	7 25 38·85	16 39·0	16 42·8	60 58·6	61 12·9
30	7 21 42·94	16 45·4	16 46·6	61 22·3	61 26·9
31	7 17 47·03	16 46·5	16 44·9	61 26·3	61 20·6

TABLE VI.—NOVEMBER, 1853.

MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Declination.	Diff. Dec. for 10m.	Hour.	Right Ascension.	Declination.	Diff. Dec. for 10m.
	THURSDAY 3.				WEDNESDAY 9.		
	h m s	° ′ ″	″		h m s	° ′ ″	″
0	16 37 9·39	S. 22 6 32·0	85·53	0	22 37 13·07	S. 14 21 11·9	120·27
1	16 39 46·87	22 15 5·2	83·92	1	22 39 18·78	14 9 10·3	120·85
2	16 42 24·60	22 23 28·7	82·30	2	22 41 24·14	13 57 5·2	121·38
3	16 45 2·58	22 31 42·5	80·65	3	22 43 29·17	13 44 56·9	121·93
4	16 47 40·80	22 39 46·4	78·98	4	22 45 33·87	13 32 45·3	122·47
5	16 50 19·27	22 47 40·3	77·33	5	22 47 38·24	13 20 30·5	122·98
6	16 52 57·96	22 55 24·3	75·67	6	22 49 42·29	13 8 12·6	123·50
7	16 55 36·88	23 2 58·3	73·95	7	22 51 46·01	12 55 51·6	123·98
8	16 58 16·02	23 10 22·0	72·27	8	22 53 49·42	12 43 27·7	124·47
9	17 0 55·36	23 17 35·6	70·55	9	22 55 52·52	12 31 0·9	124·93
10	17 3 34·91	23 24 38·9	68·83	10	22 57 55·31	12 18 31·3	125·38
11	17 6 14·66	23 31 31·9	67·08	11	22 59 57·79	12 5 59·0	125·85
12	17 8 54·59	23 38 14·4	65·35	12	23 1 59·97	11 53 23·9	126·28
13	17 11 34·71	23 44 46·5	63·58	13	23 4 1·86	11 40 46·2	126·70
14	17 14 15·00	23 51 8·0	61·83	14	23 6 3·45	11 28 6·0	127·10
15	17 16 55·45	23 57 19·0	60·05	15	23 8 4·76	11 15 23·4	127·52
16	17 19 36·06	24 3 19·3	58·27	16	23 10 5·78	11 2 38·3	127·90
17	17 22 16·82	24 9 8·9	56·48	17	23 12 6·53	10 49 50·9	128·28
18	17 24 57·72	24 14 47·8	54·68	18	23 14 7·00	10 37 1·2	128·65
19	17 27 38·75	24 20 15·9	52·88	19	23 16 7·20	10 24 9·3	129·02
20	17 30 19·90	24 25 33·2	51·07	20	23 18 7·13	10 11 15·2	129·35
21	17 33 1·17	24 30 39·6	49·25	21	23 20 6·79	9 58 19·1	129·68
22	17 35 42·54	24 35 35·1	47·43	22	23 22 6·21	9 45 21·0	130·02
23	17 38 24·01	S. 24 40 19·7	45·60	23	23 24 5·36	S. 9 32 20·9	130·33
	SUNDAY 6.				SATURDAY 12.		
0	19 48 57·95	S. 24 48 6·4	42·28	0	0 57 41·95	N. 1 24 7·2	134·07
1	19 51 32·45	24 43 52·7	43·87	1	0 59 33·83	1 37 31·6	133·92
2	19 54 6·60	24 39 29·5	45·45	2	1 1 25·66	1 50 55·1	133·77
3	19 56 40·40	24 34 56·8	47·03	3	1 3 17·46	2 4 17·7	133·62
4	19 59 13·85	24 30 14·6	48·60	4	1 5 9·23	2 17 39·4	133·45
5	20 1 46·93	24 25 23·0	50·13	5	1 7 0·97	2 31 0·1	133·27
6	20 4 19·66	24 20 22·2	51·67	6	1 8 52·68	2 44 19·7	133·08
7	20 6 52·01	24 15 12·2	53·18	7	1 10 44·37	2 57 38·2	132·90
8	20 9 23·99	24 9 53·1	54·70	8	1 12 36·04	3 10 55·6	132·70
9	20 11 55·58	24 4 24·9	56·18	9	1 14 27·70	3 24 11·8	132·50
10	20 11 26·80	23 58 47·8	57·65	10	1 16 19·35	3 37 26·8	132·27
11	20 16 57·63	23 53 1·9	59·18	11	1 18 10·99	3 50 40·4	132·07
12	20 19 28·06	23 47 7·1	60·57	12	1 20 2·62	4 3 52·8	131·83
13	20 21 58·10	23 41 8·7	62·00	13	1 21 54·26	4 17 3·8	131·58
14	20 24 27·74	23 34 51·7	63·43	14	1 23 45·90	4 30 13·3	131·35
15	20 26 56·98	23 28 31·1	64·82	15	1 25 37·54	4 43 21·4	131·08
16	20 29 25·81	23 22 2·2	66·22	16	1 27 29·20	4 56 27·9	130·83
17	20 31 54·24	23 15 24·9	67·60	17	1 29 20·87	5 9 32·9	130·55
18	20 34 22·25	23 8 39·3	68·95	18	1 31 12·56	5 22 36·2	130·28
19	20 36 49·86	23 1 45·6	70·28	19	1 33 4·27	5 35 37·9	129·98
20	20 39 17·04	22 54 43·9	71·63	20	1 34 56·01	5 48 37·8	129·70
21	20 41 43·82	22 47 34·1	72·95	21	1 36 47·77	6 1 36·0	129·40
22	20 44 10·17	22 40 16·4	74·23	22	1 38 39·57	6 14 32·4	129·08
23	20 46 36·09	22 32 51·0	75·53	23	1 40 31·40	6 27 26·9	128·77
24	20 49 1·60	S. 22 25 17·8		24	1 42 23·27	N. 6 40 19·5	

TABLE VI. (Continued.)—NOVEMBER, 1853.

MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Declination.	Diff. Dec. for 10m.	Hour.	Right Ascension.	Declination.	Diff. Dec. for 10m.
	TUESDAY 15.				MONDAY 21.		
	h m s	° ′ ″	″		h m s	° ′ ″	″
0	3 13 36·99	N. 16 5 42·2	103·62	0	8 18 49·00	N. 23 51 39·1	51·78
1	3 15 34·23	16 16 3·9	102·92	1	8 20 58·64	23 46 28·4	52·95
2	3 17 31·64	16 26 21·4	102·20	2	8 23 8·18	23 41 10·7	54·10
3	3 19 29·22	16 36 34·6	101·48	3	8 25 17·62	23 35 46·1	55·25
4	3 21 26·96	16 46 43·5	100·75	4	8 27 26·96	23 30 14·6	56·40
5	3 23 24·87	16 56 48·0	100·02	5	8 29 36·19	23 24 36·2	57·53
6	3 25 22·96	17 6 48·1	99·27	6	8 31 45·32	23 18 51·0	58·68
7	3 27 21·21	17 16 43·7	98·52	7	8 33 54·35	23 12 58·9	59·80
8	3 29 19·64	17 26 34·8	97·75	8	8 36 3·27	23 7 0·1	60·93
9	3 31 18·24	17 36 21·3	96·98	9	8 38 12·08	23 0 54·5	62·07
10	3 33 17·01	17 46 3·2	96·20	10	8 40 20·78	22 54 42·1	63·18
11	3 35 15·97	17 55 40·4	95·43	11	8 42 29·37	22 48 23·0	64·30
12	3 37 15·10	18 5 13·0	94·63	12	8 44 37·85	22 41 57·2	65·42
13	3 39 14·41	18 14 40·8	93·85	13	8 46 46·21	22 35 24·7	66·52
14	3 41 13·90	18 24 3·9	93·03	14	8 48 54·46	22 28 45·6	67·63
15	3 43 13·57	18 33 22·1	92·22	15	8 51 2·60	22 21 59·8	68·73
16	3 45 13·42	18 42 35·4	91·42	16	8 53 10·62	22 15 7·4	69·82
17	3 47 13·45	18 51 43·9	90·57	17	8 55 18·52	22 8 8·5	70·92
18	3 49 13·66	19 0 47·3	89·73	18	8 57 26·31	22 1 3·0	72·00
19	3 51 14·06	19 9 45·7	88·90	19	8 59 33·98	21 53 51·0	73·07
20	3 53 14·64	19 18 39·1	88·05	20	9 1 41·54	21 46 32·6	74·15
21	3 55 15·40	19 27 27·4	87·18	21	9 3 48·97	21 39 7·7	75·23
22	3 57 16·34	19 36 10·5	86·33	22	9 5 56·29	21 31 36·3	76·28
23	3 59 17·47	N. 19 44 48·5	85·45	23	9 8 3·48	N. 21 23 58·6	77·35
	FRIDAY 18.				THURSDAY 24.		
0	5 41 48·11	N. 24 48 3·9	34·95	0	10 49 43·31	N. 13 11 13·0	122·23
1	5 43 57·46	24 51 33·6	33·80	1	10 51 45·72	12 58 59·6	123·00
2	5 46 6·92	24 54 56·4	32·63	2	10 53 48·08	12 46 41·6	123·75
3	5 48 16·50	24 58 12·2	31·47	3	10 55 50·41	12 34 19·1	124·47
4	5 50 26·19	25 1 21·0	30·28	4	10 57 52·69	12 21 52·3	125·22
5	5 52 35·99	25 4 22·7	29·12	5	10 59 54·94	12 9 21·0	125·93
6	5 54 45·89	25 7 17·4	27·95	6	11 1 57·16	11 56 45·4	126·65
7	5 56 55·90	25 10 5·1	26·75	7	11 3 59·35	11 44 5·5	127·37
8	5 59 6·01	25 12 45·6	25·57	8	11 6 1·51	11 31 21·3	128·05
9	6 1 16·21	25 15 19·0	24·38	9	11 8 3·65	11 18 33·0	128·75
10	6 3 26·50	25 17 45·3	23·18	10	11 10 5·77	11 5 40·5	129·45
11	6 5 36·89	25 20 4·4	22·00	11	11 12 7·88	10 52 43·8	130·12
12	6 7 47·36	25 22 16·4	20·80	12	11 14 9·97	10 39 43·1	130·78
13	6 9 57·92	25 24 21·2	19·60	13	11 16 12·05	10 26 38·4	131·45
14	6 12 8·55	25 26 18·8	18·38	14	11 18 14·13	10 13 29·7	132·10
15	6 14 19·26	25 28 9·1	17·20	15	11 20 16·21	10 0 17·1	132·75
16	6 16 30·04	25 29 52·3	15·98	16	11 22 18·29	9 47 0·6	133·38
17	6 18 40·90	25 31 28·2	14·77	17	11 24 20·37	9 33 40·3	134·02
18	6 20 51·82	25 32 56·8	13·57	18	11 26 22·46	9 20 16·2	134·63
19	6 23 2·80	25 34 18·2	12·35	19	11 28 24·56	9 6 48·4	135·25
20	6 25 13·85	25 35 32·3	11·15	20	11 30 26·68	8 53 16·9	135·85
21	6 27 24·95	25 36 39·2	9·92	21	11 32 28·82	8 39 41·8	136·45
22	6 29 36·11	25 37 38·7	8·70	22	11 34 30·99	8 26 3·1	137·03
23	6 31 47·32	25 38 30·9	7·48	23	11 36 33·18	8 12 20·9	137·62
24	6 33 58·57	N. 25 39 15·8		24	11 38 35·41	N. 7 58 35·2	

TABLE VI. (Continued.)—NOVEMBER, 1853.

MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Declination.	Diff. Dec. for 10m.	Hour.	Right Ascension.	Declination.	Diff. Dec. for 10m.
	SUNDAY 27.				WEDNESDAY 30.		
	h m s	° ′ ″	″		h m s	° ′ ″	″
0	13 18 24·23	S. 3 50 13·5	152·98	0	16 8 25·89	S. 20 22 13·1	104·40
1	13 20 33·42	4 5 31·4	152·98	1	16 11 2·48	20 32 39·5	102·95
2	13 22 42·86	4 20 49·3	152·97	2	16 13 39·47	20 42 57·2	101·48
3	13 24 52·57	4 36 7·1	152·93	3	16 16 16·86	20 53 6·1	100·02
4	13 27 2·54	4 51 24·7	152·87	4	16 18 54·64	21 3 6·2	98·53
5	13 29 12·80	5 6 41·9	152·80	5	16 21 32·81	21 12 57·4	97·00
6	13 31 23·33	5 21 58·7	152·72	6	16 24 11·36	21 22 39·4	95·47
7	13 33 34·14	5 37 15·0	152·63	7	16 26 50·29	21 32 12·2	93·90
8	13 35 45·24	5 52 30·8	152·50	8	16 29 29·60	21 41 35·6	92·33
9	13 37 56·64	6 7 45·8	152·35	9	16 32 9·27	21 50 49·6	90·75
10	13 40 8·33	6 22 59·9	152·22	10	16 34 49·30	21 59 54·1	89·13
11	13 42 20·33	6 38 13·2	152·05	11	16 37 29·69	22 8 48·9	87·50
12	13 44 32·63	6 53 25·5	151·87	12	16 40 10·43	22 17 33·9	85·85
13	13 46 45·24	7 8 36·7	151·65	13	16 42 51·52	22 26 9·0	84·20
14	13 48 58·17	7 23 46·6	151·45	14	16 45 32·94	22 34 34·2	82·52
15	13 51 11·42	7 38 55·3	151·20	15	16 48 14·69	22 42 49·3	80·82
16	13 53 25·00	7 54 2·5	150·95	16	16 50 56·76	22 50 54·2	79·10
17	13 55 38·90	8 9 8·2	150·68	17	16 53 39·15	22 58 48·8	77·37
18	13 57 53·14	8 24 12·3	150·38	18	16 56 21·84	23 6 33·0	75·63
19	14 0 7·71	8 39 14·6	150·07	19	16 59 4·84	23 14 6·8	73·87
20	14 2 22·63	8 54 15·0	149·75	20	17 1 48·12	23 21 30·0	72·10
21	14 4 37·89	9 9 13·5	149·40	21	17 4 31·69	23 28 42·6	70·30
22	14 6 53·51	9 24 9·9	149·05	22	17 7 15·53	23 35 44·4	68·52
23	14 9 9·47	S. 9 39 4·2	148·65	23	17 9 59·64	23 42 35·5	66·68
				24	17 12 44·00	S. 23 49 15·6	

TABLE IX.—NOVEMBER.

ECLIPSES OF JUPITER'S SATELLITES.

MEAN TIME.

			d	h	m	s				d	h	m	s
I.	Eclipse	Reapp.	1	1	31	9·4	I.	Eclipse	Reapp.	15	5	20	23·3
II.	Eclipse	Reapp.	2	18	46	27·4	I.	Eclipse	Reapp.	16	23	49	0·7
I.	Eclipse	Reapp.	2	19	59	48·8	II.	Eclipse	Reapp.	17	0	2	1·7
I.	Eclipse	Reapp.	4	14	28	29·8	I.	Eclipse	Reapp.	18	18	17	39·3
II.	Eclipse	Reapp.	6	8	5	55·6	I.	Eclipse	Reapp.	20	12	46	17·7
I.	Eclipse	Reapp.	6	8	57	9·7	II.	Eclipse	Reapp.	20	13	21	26·3
III.	Eclipse	Disapp.	7	22	8	14·0	I.	Eclipse	Reapp.	22	7	14	54·2
III.	Eclipse	Reapp.	8	0	59	0·1	III.	Eclipse	Reapp.	22	8	58	53·9
I.	Eclipse	Reapp.	8	3	25	48·5	I.	Eclipse	Reapp.	24	1	43	30·5
II.	Eclipse	Reapp.	9	21	24	16·3	II.	Eclipse	Reapp.	24	2	39	42·4
I.	Eclipse	Reapp.	9	21	54	26·8	I.	Eclipse	Reapp.	25	20	12	7·7
I.	Eclipse	Reapp.	11	16	23	6·5	I.	Eclipse	Reapp.	27	14	40	45·2
II.	Eclipse	Reapp.	13	10	43	43·2	II.	Eclipse	Reapp.	27	15	59	4·3
I.	Eclipse	Reapp.	13	10	51	45·7	I.	Eclipse	Reapp.	29	9	9	20·5
III.	Eclipse	Reapp.	15	4	58	48·1	III.	Eclipse	Reapp.	29	12	58	54·9

TABLE VII.—NOVEMBER, 1853.

MEAN TIME.

LUNAR DISTANCES.

Day of the Month.	Star's Name and Position.		*Noon.*	$\mathrm{III}^{\mathrm{h}}$.	VI^{h}.	IX^{h}.
3	Sun . . .	W.	29° 51′ 50″	31° 36′ 10″	33° 20′ 24″	35° 4′ 31″
	Fomalhaut .	E.	81 32 1	79 44 10	77 56 30	76 9 3
	α Pegasi . .	E.	99 58 21	98 16 29	96 34 39	94 52 54
6	Sun . . .	W.	70 37 26	72 15 41	73 53 35	75 31 9
	Venus . .	W.	27 21 16	28 58 20	30 35 4	32 11 29
	α Arietis . .	E.	101 27 57	99 44 52	98 2 6	96 19 39
9	Sun . . .	W.	108 23 45	109 54 19	111 24 36	112 54 38
	α Aquilæ .	W.	48 43 43	49 51 35	51 0 27	52 10 15
	Aldebaran .	E.	91 42 57	90 5 12	88 27 43	86 50 31
12	α Aquilæ .	W.	78 26 18	79 43 27	81 0 37	82 17 46
	α Pegasi . .	W.	32 25 26	33 25 17	34 27 13	35 31 0
	Aldebaran .	E.	53 43 11	52 10 49	50 38 38	49 6 38
15	Pollux . .	E.	61 19 34	59 49 48	58 20 8	56 50 33
	Regulus . .	E.	97 27 33	95 57 54	94 28 20	92 58 52
	Mars . . .	E.	99 4 10	97 37 54	96 11 43	94 45 37
18	α Arietis .	W.	50 39 52	52 4 48	53 29 53	54 55 6
	Aldebaran .	W.	19 21 53	20 47 23	22 13 17	23 39 31
	Regulus . .	E.	61 55 16	60 27 0	58 58 44	57 30 29
21	Regulus . .	E.	26 33 10	25 4 26	23 35 44	22 7 6
	Spica ♍ . .	E.	80 30 20	79 0 30	77 30 30	76 0 21
	Sun . . .	E.	117 35 33	116 12 46	114 49 50	113 26 43
24	Regulus . .	W.	11 59 8	13 28 56	15 0 30	16 33 29
	Spica ♍ . .	E.	43 35 22	41 59 45	40 23 50	38 47 38
	Sun . . .	E.	83 25 22	81 56 9	80 26 34	78 56 36
27	Pollux . .	W.	87 53 0	89 39 59	91 27 24	93 15 15
	Regulus . .	W.	51 49 37	53 36 11	55 23 13	57 10 43
	Sun . . .	E.	45 48 40	44 9 6	42 29 5	40 48 38

TABLE X.—Right Ascension of the Planet Jupiter at Transit over the Meridian of Greenwich, November 1853.

Day.	Æ.	Day.	Æ.	Day.	Æ.
	h m s		h m s		h m s
1	17 32 19·09	11	17 40 59·05	21	17 50 7·90
2	17 33 9·59	12	17 41 52·74	22	17 51 4·14
3	17 34 0·44	13	17 42 46·71	23	17 52 0·60
4	17 34 51·64	14	17 43 40·96	24	17 52 57·28
5	17 35 43·18	15	17 44 35·47	25	17 53 54·17
6	17 36 35·04	16	17 45 30·25	26	17 54 51·27
7	17 37 27·22	17	17 46 25·29	27	17 55 48·57
8	17 38 19·72	18	17 47 20·58	28	17 56 46·06
9	17 39 12·53	19	17 48 16·11	29	17 57 43·73
10	17 40 5·64	20	17 49 11·89	30	17 58 41·59

TABLE VII.—NOVEMBER, 1853.

MEAN TIME.

LUNAR DISTANCES.

Day of the Month.	Star's Name and Position.		*Midnight.*	XV^{h}.	XVIII^{h}.	XXI^{h}.
3	Sun . . .	W.	36° 48′ 29″	38° 32′ 18″	40° 15′ 56″	41° 59′ 23″
	Fomalhaut .	E.	74 21 50	72 34 53	70 48 13	69 1 51
	α Pegasi . .	E.	93 11 15	91 32 43	89 48 22	88 7 12
6	Sun . . .	W.	77 8 22	78 45 15	80 21 48	81 58 0
	Venus . .	W.	33 47 33	35 23 16	36 58 40	38 33 43
	α Arietis .	E.	94 37 30	92 55 41	91 14 12	89 33 2
9	Sun . . .	W.	114 24 24	115 53 55	117 23 10	118 52 10
	α Aquilæ .	W.	53 20 54	54 32 19	55 44 25	56 57 9
	Aldebaran .	E.	85 13 34	83 36 52	82 0 26	80 24 15
12	α Aquilæ .	W.	83 34 53	84 51 58	86 9 0	87 25 57
	α Pegasi . .	W.	36 36 28	37 43 28	38 51 49	40 1 24
	Aldebaran .	E.	47 34 49	46 3 11	44 31 45	43 0 29
15	Pollux . .	E.	55 21 4	53 51 40	52 22 22	50 53 8
	Regulus . .	E.	91 29 28	90 0 10	88 30 56	87 1 47
	Mars . . .	E.	93 19 36	91 53 39	90 27 47	89 1 59
18	α Arietis .	W.	56 20 26	57 45 54	59 11 29	60 37 11
	Aldebaran .	W.	25 6 1	26 32 45	27 59 42	29 26 50
	Regulus . .	E.	56 2 14	54 34 0	53 5 45	51 37 30
21	Regulus . .	E.	20 38 34	19 10 12	17 42 4	16 14 18
	Spica ♍ . .	E.	74 30 1	72 59 30	71 28 48	69 57 54
	Sun . . .	E.	112 3 26	110 39 57	109 16 16	107 52 22
24	Regulus . .	W.	18 7 37	19 42 44	21 18 43	22 55 29
	Spica ♍ . .	E.	37 11 8	35 34 21	33 57 18	32 20 0
	Sun . . .	E.	77 26 15	75 55 29	74 24 20	72 52 45
27	Pollux . .	W.	95 3 31	96 52 12	98 41 18	100 30 47
	Regulus . .	W.	58 58 39	60 47 1	62 35 48	64 25 0
	Sun . . .	E.	39 7 46	37 26 29	35 44 47	34 2 42

TABLE XI.—Dip of the Sea Horizon.

(Calculated by Formula, p. 90.)

Height.	Dip.	Height.	Dip.	Height.	Dip.
Feet.	′ ″	Feet.	′ ″	Feet.	′ ″
5	2 13	12	3 26	35	5 52
6	2 26	13	3 35	40	6 17
7	2 38	14	3 43	50	7 1
8	2 49	15	3 51	60	7 42
9	2 59	20	4 26	70	8 18
10	3 8	25	4 58	80	8 53
11	3 17	30	5 26	90	9 25

TABLE VIII.—MOON-CULMINATING STARS.

Date.	Name.	At Greenwich Transit.		
		Apparent Right Ascension *in Time.*	Var. of ☾'s R. A. in 1 hour of Long.	Sidereal Time of ☾'s Sem. pas. mer.
1853.		h m s	s	s
Nov. 1	Moon I.	14 35 59·42	146·80	70·78
2	Moon I.	15 36 33·50	156·12	73·13
3	Moon I.	16 40 45·94	164·51	75·22
4	Moon I.	17 47 37·80	168·89	76·32
5	μ^1 Sagittarii	18 4 59·08		
	λ Sagittarii	18 18 54·79		
	Moon I.	18 55 2·40	167·05	75·93
	h^2 Sagittarii	19 27 46·62		
	b Sagittarii	19 47 56·58		
6	h^2 Sagittarii	19 27 46·61		
	b Sagittarii	19 47 56·56		
	Moon I.	20 0 28·87	159·34	74·11
	ρ Capricorni	20 20 29·61		
	ψ Capricorni	20 37 24·66		
7	ρ Capricorni	20 20 29·59		
	ψ Capricorni	20 37 24·64		
	Moon I.	21 2 4·50	148·35	71·40
	δ Capricorni	21 38 57·11		
	μ Capricorni	21 45 18·47		
8	δ Capricorni	21 38 57·09		
	μ Capricorni	21 45 18·46		
	Moon I.	21 59 7·49	137·06	68·49
	σ Aquarii	22 22 53·79		
	δ Aquarii	22 46 52·73		
9	σ Aquarii	22 22 53·78		
	δ Aquarii	22 46 52·71		
	Moon I.	22 51 57·30	127·48	65·92
	φ Aquarii	23 6 44·69		
	ψ^3 Aquarii	23 11 20·85		
10	φ Aquarii	23 6 44·68		
	ψ^3 Aquarii	23 11 20·84		
	Moon I.	23 41 27·75	120·53	63·97
	27 Piscium	23 51 11·10		
	33 Piscium	23 57 50·98		
11	27 Piscium	23 51 11·09		
	33 Piscium	23 57 50·97		
	Moon I.	0 28 44·99	116·38	62·75
	δ Piscium	0 41 6·05		
	20 Ceti	0 45 32·32		
12	δ Piscium	0 41 6·05		

TABLE VIII. (Continued.)

MOON-CULMINATING STARS.

Date.	Name.	At Greenwich Transit.		
		Apparent Right Ascension *in Time.*	Var. of ☾'s R. A. in 1 hour of Long.	Sidereal Time of ☾'s Sem. pas. mer.
1853.		h m s	s	s
Nov. 12	20 Ceti …………	0 45 32·31		
	Moon I. ……	1 14 55·20	114·90	62·27
	ν Piscium ……	1 33 49·77		
	o Piscium ……	1 37 40·93		
13	ν Piscium ……	1 33 49·77		
	o Piscium ……	1 37 40·93		
	Moon I. ……	2 0 59·06	115·79	62·46
	ξ^2 Ceti …………	2 20 23·85		
14	ξ^2 Ceti …………	2 20 23·85		
	Moon I. ……	2 47 48·88	118·65	63·22
	δ Arietis ………	3 3 16·89		
	τ^1 Arietis ………	3 12 47·93		
15	δ Arietis ………	3 3 16·90		
	τ^1 Arietis ………	3 12 47·94		
	Moon II. ……	3 38 14·04	123·00	64·37
	γ Tauri…………	4 11 29·14		
	δ^2 Tauri…………	4 15 40·75		
16	γ Tauri…………	4 11 29·16		
	δ^2 Tauri…………	4 15 40·77		
	Moon II. ……	4 28 24·02	127·85	65·71
	ι Tauri ………	4 54 21·99		
	n Tauri ………	5 10 30·04		
17	ι Tauri ………	4 54 22·01		
	n Tauri ………	5 10 30·06		
	Moon II. ……	5 20 27·41	132·27	66·94
	132 Tauri…………	5 40 2·76		
	136 Tauri…………	5 44 8·66		
18	132 Tauri…………	5 40 2·79		
	136 Tauri…………	5 44 8·69		
	Moon II. ……	6 14 1·60	135·26	67·80
	ϵ Geminor. ……	6 34 56·36		
	ζ Geminor. ……	6 55 26·22		
19	ϵ Geminor. ……	6 34 56·39		
	ζ Geminor. ……	6 55 26·25		
	Moon II. ……	7 8 22·98	136·14	68·12
	κ Geminor. ……	7 35 36·87		
	ϕ Geminor. ……	7 44 32·55		
20	κ Geminor. ……	7 35 36·90		
	ϕ Geminor. ……	7 44 32·58		
	Moon II. ……	8 2 39·17	134·89	67·87
	θ Cancri ………	8 23 14·99		
	γ Cancri ………	8 34 48·90		

TABLE VIII. (Continued.)

MOON-CULMINATING STARS.

Date.	Name.	At Greenwich Transit.		
		Apparent Right Ascension *in Time*.	Var. of ☾'s R. A. in 1 hour of Long.	Sidereal Time of ☾'s Sem. pas. mer.
1853.		h m s	s	s
Nov. 21	θ Canceri............	8 23 15·02		
	γ Canceri............	8 34 48·94		
	Moon II.	8 56 5·75	132·17	67·22
	λ Leonis	9 23 21·76		
	ψ Leonis	9 35 45·22		
22	λ Leonis	9 23 21·80		
	ψ Leonis	9 35 45·26		
	Moon II.	9 48 20·01	129·05	66·43
	α Leonis	10 0 34·17		
	γ Leonis	10 11 53·58		
23	α Leonis	10 0 34·20		
	γ Leonis	10 11 53·61		
	Moon II.	10 39 26·43	126·69	65·79
	χ Leonis	10 57 27·43		
	ι Leonis	11 16 16·66		
24	χ Leonis	10 57 27·46		
	ι Leonis	11 16 16·69		
	Moon II.	11 29 55·10	126·05	65·60
	π Virginis.........	11 53 21·76		
	ο Virginis.........	11 57 44·48		
25	π Virginis.........	11 53 21·79		
	ο Virginis.........	11 57 44·51		
	Moon II.	12 20 36·67	127·88	66·03
	δ Virginis	12 48 12·85		
	θ Virginis	13 2 21·38		
26	Moon II.	13 12 37·18	132·69	67·25
27	Moon II.	14 7 11·26	140·67	69·25
28	Moon II.	15 5 31·00	151·33	71·88
29	Moon II.	16 8 22·80	162·92	74·68

FINIS.

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